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ABSTRACT

This is one of eighteen sets of individualized mathematics problems developed by the Oregon Vo-Tech Math Project. Each of these problem packages is organized around a mathematical topic and contains problems related to diverse vocations. Solutions are provided for all problems. Problems in this volume require the use of trigonometric and inverse trigonometric functions. They are drawn from eleven vocational areas: machine tools, drafting, industrial mechanics, auto mechanics, aviation mechanics, forest products, forestry, fire and police science, diesel mechanics, electronics, and agriculture. (SD)

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I
L INDIVIDUALIZED
S LEARNING SYSTEMS

Individualized Math Problems in

Trigonometry

Oregon Vo-Tech Mathematics Problem Sets

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SE 020 646

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Individualized Learning Systems

MATHEMATICS

Trigonometry

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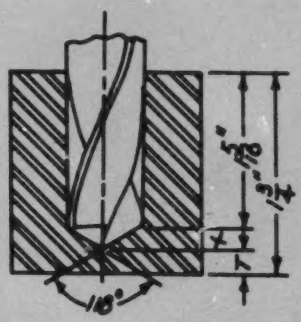
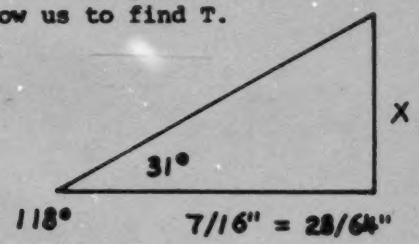
A. Problem with Complete Solution

1. To dimension a blind "hole", the draftsman shows depth to the beginning of the conical surface, 1 5/16 in. It is often necessary to compute the thickness of metal, T. What is T to the nearest 64th of an inch if the drill is 7/8" in diameter?



Solution:

Need to find distance X and add to length of bit, then subtracting from total depth of metal, we have T. Using idea of supplementary angles (a straight angle = 180°), we can find acute angle of triangle. Then using the definition of tangent, we can find height (X). Which will allow us to find T.



$180^\circ - 118^\circ = 62^\circ$

$\frac{X}{28/64"} = \tan 31^\circ$

$X = (28/64")(0.60086)$

$X = 0.26288" \approx 17/64$ in

$T = 1\ 3/4" - (X + 1\ 5/16")$

$T = 112/64" - (17/64" + 1\ 20/64")$

$T = 112/64" - (17/64" + 84/64")$

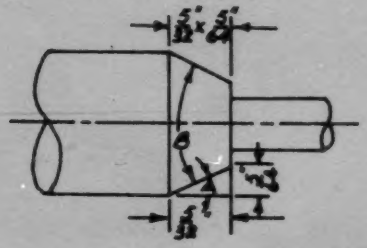
$T = 112/64" - 101/64"$

$T = 11/64$ in

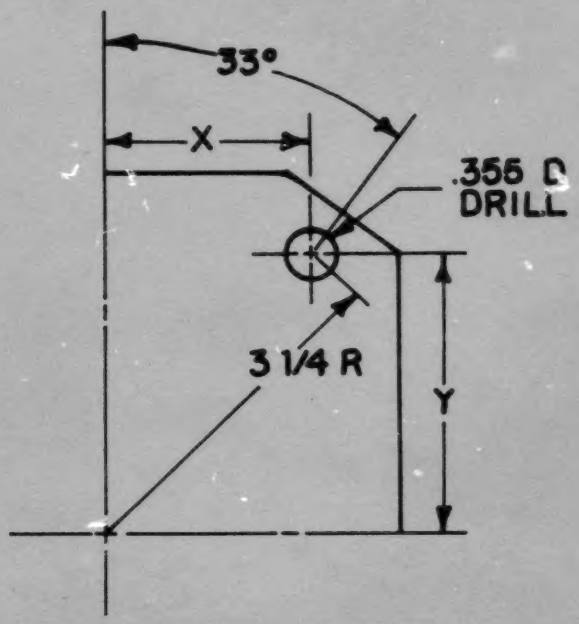
B. Problems without Complete Solutions

2. The dimensions at the top are the only ones given by the draftsman. To turn angle B, the machinist must find angle A. Find angle A to nearest half degree.

Solution:



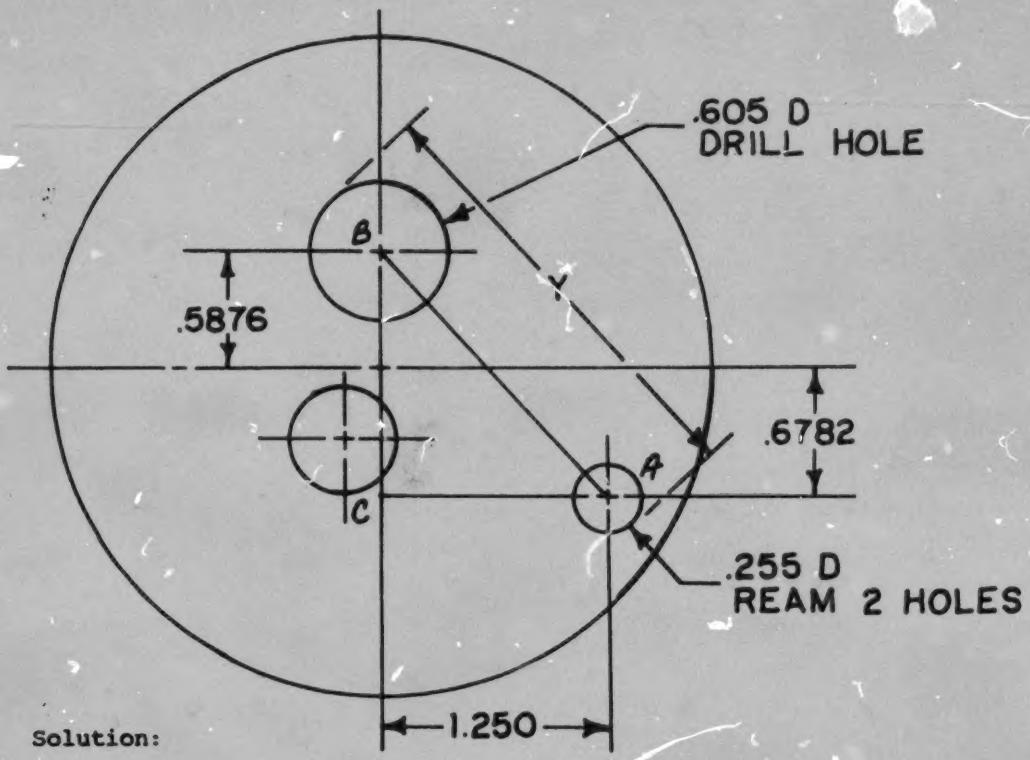
3. To locate the 0.355 diameter hole, it is necessary to determine dimensions from the vertical and horizontal centerlines. Determine these dimensions.



Solution:

B. Problems without Complete Solutions (continued)

4. The location of the 0.605 diameter drill hole and the 0.255 reamed hole is checked by inserting pins of the same size in the respective hole and then measuring over them with a micrometer. What should the micrometer reading (Y) be if the holes are properly located?



Solution:

Values of Trigonometric Functions
Angle in Degrees

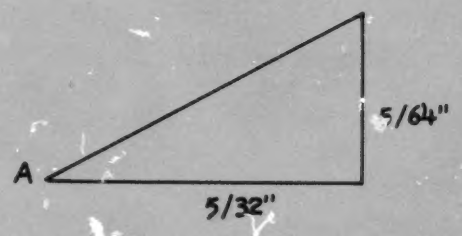
Degrees	Sin	Cos	Tan	Cot	Sec	Csc
26° 00'	.4384	.8988	.4877	2.050	1.113	2.281
10'	.4410	.8975	.4913	2.035	1.114	2.268
20'	.4436	.8962	.4950	2.020	1.116	2.254
30'	.4462	.8949	.4986	2.006	1.117	2.241
40'	.4488	.8936	.5022	1.991	1.119	2.228
50'	.4514	.8923	.5059	1.977	1.121	2.215
27° 00'	.4540	.8910	.5095	1.963	1.122	2.203
30° 00'	.50000	.86603	.57735	1.7321	1.1547	2.0000
31° 00'	.51504	.85717	.60086	1.6643	1.1666	1.9316
32° 00'	.52992	.84805	.62487	1.5003	1.1779	1.8871
45° 00'	.7071	.7071	1.00000	1.0000	1.4142	1.4142
10'	.7092	.7050	1.0058	.9942	1.4184	1.4100
20'	.7112	.7030	1.0117	.9884	1.4225	1.4061
30'	.7133	.7009	1.0176	.9827	1.4267	1.4019
40'	.7153	.6988	1.0235	.9770	1.4310	1.3980
50'	.7173	.6967	1.0295	.9713	1.4353	1.3941
46° 00'	.7193	.6947	1.0355	.9657	1.4396	1.3802
56° 00'	.81904	.55919	1.4826	.6745	1.7882	1.1792
57° 00'	.83867	.54464	1.5399	.6494	1.8361	1.1924
58° 00'	.84805	.52992	1.6003	.6249	1.8871	1.2062

2. $\tan A = \frac{5/64 \text{ in}}{5/32 \text{ in}}$

$= (5/64)(32/5)$

$= 0.5000$

$A = 26^\circ\ 30'$



3. Horizontal distance = X

Vertical distance = Y

angle A = $90^\circ - 33^\circ = 57^\circ$

$\frac{Y}{3.25"} = \sin 57^\circ$

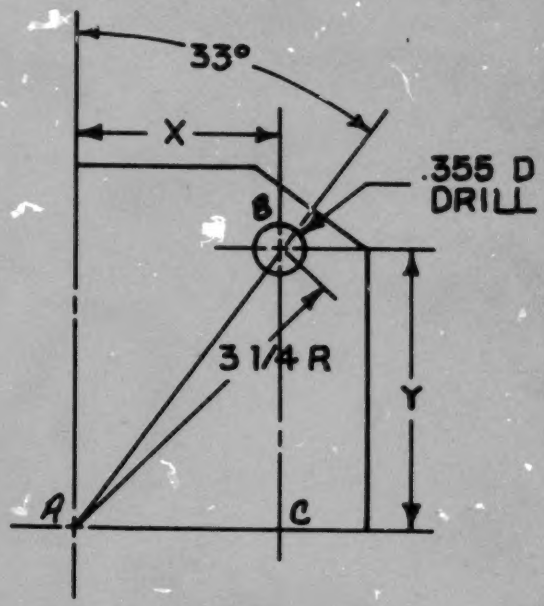
$Y = (3.25 \text{ in})(0.83867)$

$Y = 2.7257 \text{ in}$

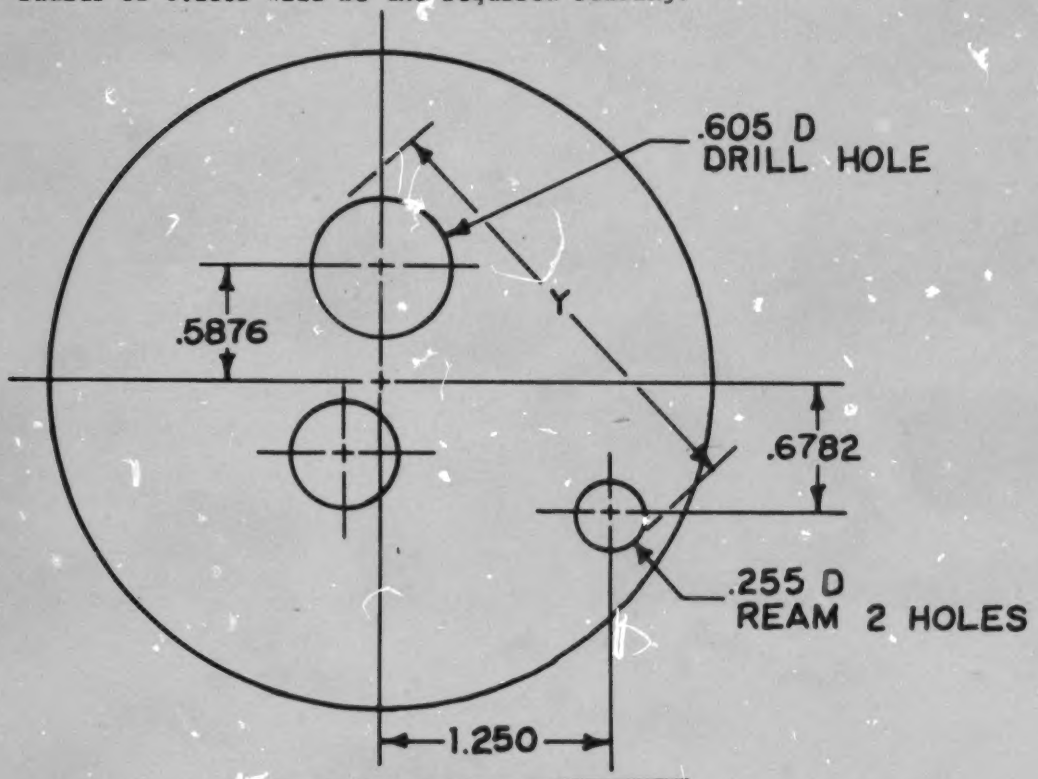
$\frac{X}{3.25"} = \cos 57^\circ$

$X = (3.25 \text{ in})(0.54464)$

$X = 1.77008 \text{ in}$



4. Draw in a right triangle and find the length of the hypotenuse. The total of the hypotenuse and the radius of 0.605D and the radius of 0.255D will be the required reading.



$$Hyp = \sqrt{(1.250")^2 + (0.5876" + 0.6782")^2}$$

$$Hyp = \sqrt{(1.250")^2 + (1.2658")^2}$$

$$Hyp = \sqrt{1.5625 \text{ sq in} + 1.6022 \text{ sq in}}$$

$$Hyp = \sqrt{3.1647 \text{ sq in}}$$

$$Hyp = 1.77897 \text{ in}$$

$$Y = 1.77897" + 0.3025" + 0.1275"$$

$$Y = 2.20897 \text{ in} \quad \text{Micrometer reading}$$

Complete Solutions to 8 problems (continued)

4. Alternate Solution:

$$\tan A = \frac{1.2658 \text{ in}}{1.250 \text{ in}}$$

$$= 1.01264$$

$$A \approx 45^\circ\ 22'$$

$$\frac{1.2658 \text{ in}}{C} = \sin 45^\circ\ 22'$$

$$C = \frac{1.2658 \text{ in}}{0.71162}$$

$$C = 1.77876 \text{ in}$$

$$Y = 1.77876" + 0.3025" + 0.1275"$$

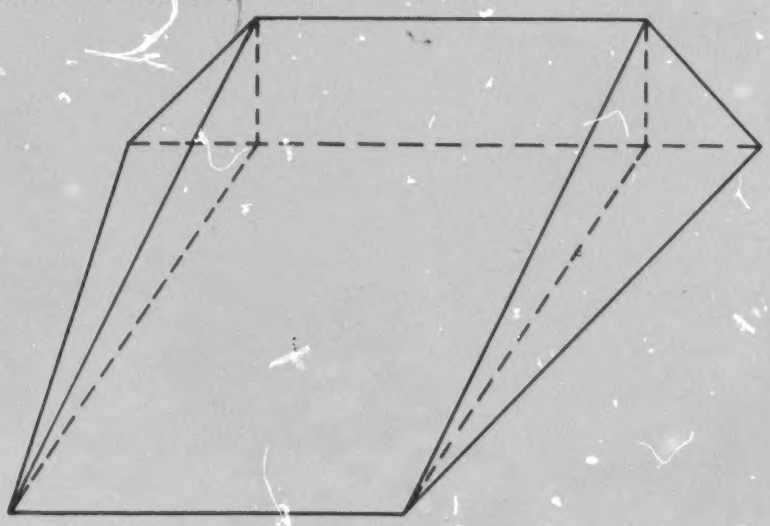
$$Y = 2.20076 \text{ in}$$

A. Problems with solutions.

1. A particular bridge approach is 125' long, 30' wide and has an incline of 15%. The banks of this approach drop away with a 1:1 slope.

Problem:

How many cubic yards (cu yds) of fill are required for this approach? Assume 10% compaction.



Solution:

To find the volume of the entire approach, first find the volume of the center wedge, then the two outer pyramidal forms. To find the area of either you must find the height of the end of the approach and the length of the base.

The slope is 15%. Therefore, the height of the wedge is

$(125')(\sin 8.53^{\circ}) = (125')(.15) = 18.54'$

The base of the wedge (and the height of the pyramid) is

$(125')(\cos 8.53^{\circ}) = (125')\ (.988) = 123.62'$

Volume of the wedge is

$(1/2\ bh)(T) = 1/2\ (123.62')(18.54')(30')$

$= 34377.9$ cu ft

Volume of the pyramids is

$$2\left[1/3\ bh\right] = 2\left[1/3\left(\frac{(18.54')(18.54')}{2}\right)\right](3.23.62')$$

$= 2\ (7082.02$ cu ft.$)$

$= 14164.03$ cu ft

A. Problems with solutions.

1. continued.

So, the total volume to be filled is

34,377.9 cu ft + 14,164.03 cu ft = 48541.93 cu ft

Since there is 10% compaction

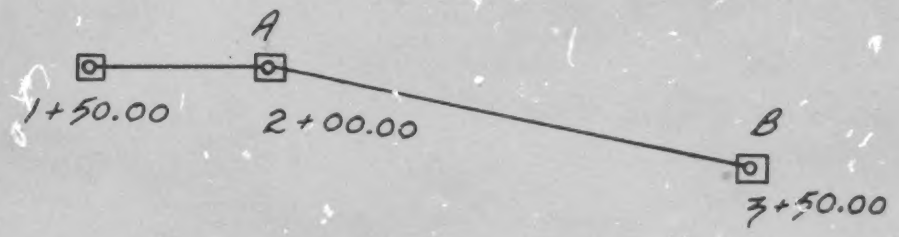
(90%)(N cu ft rock) = 48541.93 cu ft

(.90)(N) = 48541.93 cu ft

$$N = \frac{48541.93 \text{ cu ft}}{.90}$$

= 53935.48 cu ft

2.

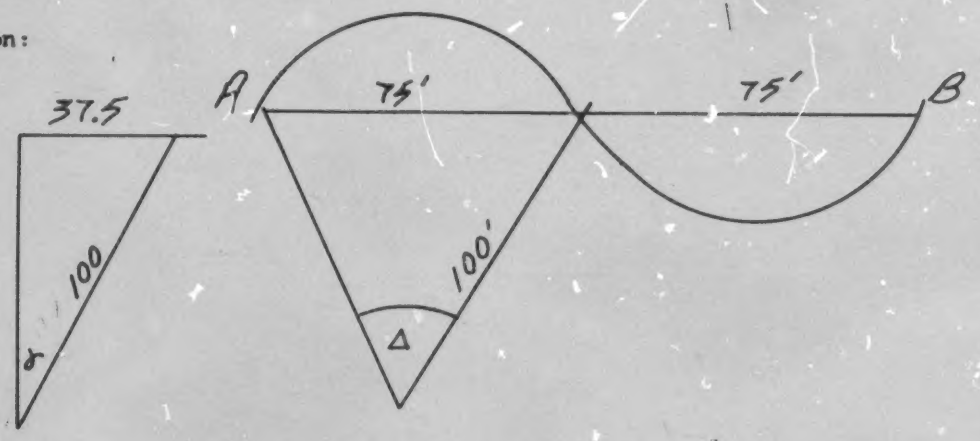


A symmetrical S shaped curve is to be laid out between points A and B. The radius to be 100'.

Problem: Determine the length of the long chord and the delta angle.

Note: The delta angle is the angle between the two projected radii.

Solution:



Since the separation is 35 0' - 200' = 150', the longest chord must be:

$$\frac{150'}{2} = 75' \text{ long}$$

A. Problems with solutions.

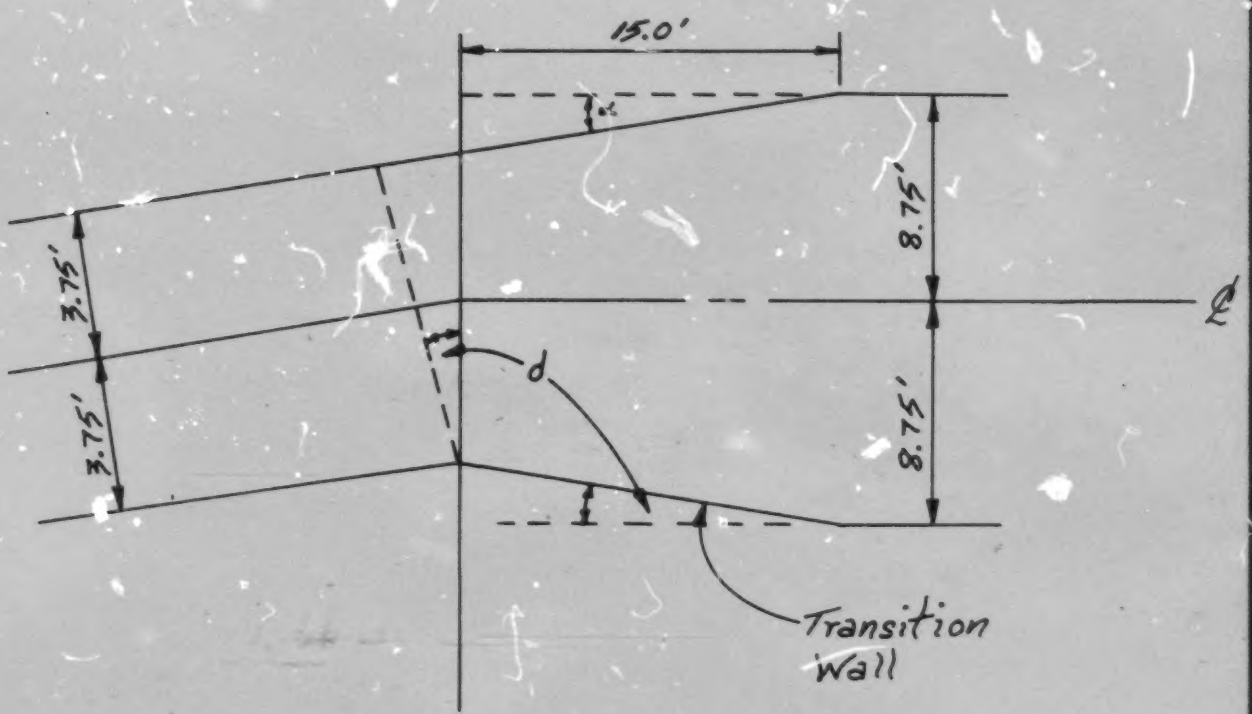
2. continued. From the drawing: $\sin\alpha = \frac{37.5'}{100'} = .375$

$$\text{Arc sin } \alpha = 22.024^\circ$$

$$\text{Therefore, } \Delta = 44.048^\circ$$

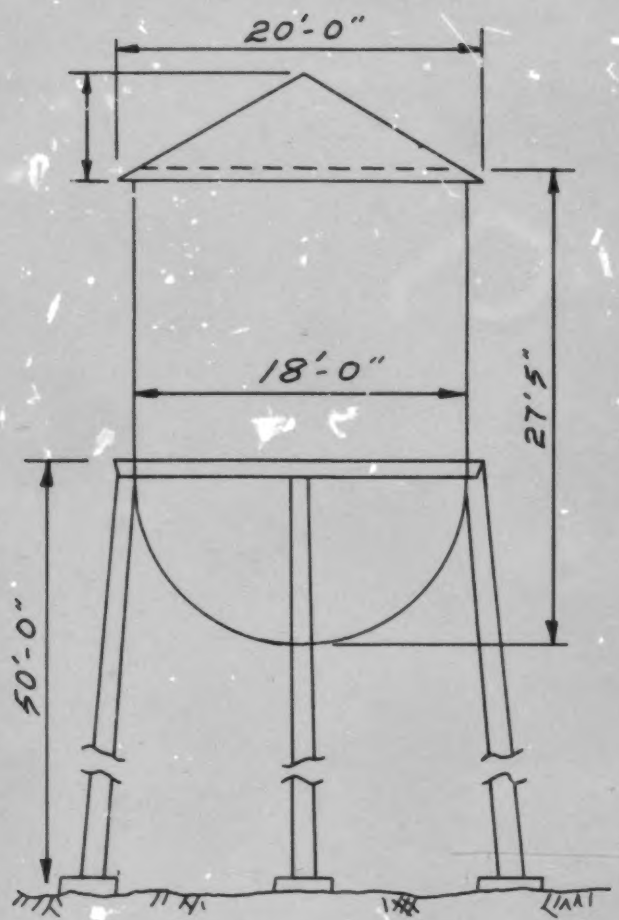
B. Problems without solutions.

3. Problem: A 17.5' flume transcends to a 7.5' flume as shown below. Find α such that the angle of transition is equal for given desired dimensions. This angle must be the same in order to provide uniform water flow.



B. Problems without solutions.

4.



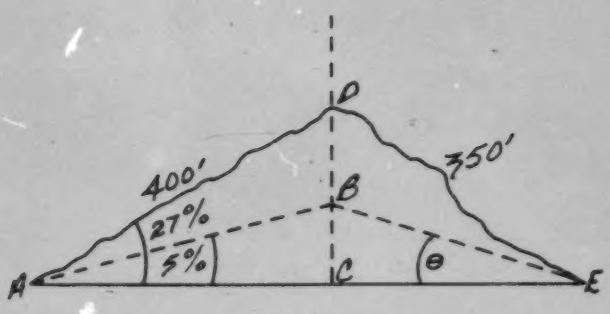
Data: Hemispherical bottom elevated water tank as dimensioned.
Tank made of 3/4" steel plate, supported on 4 steel columns.
Tank is part of the municipal water system that serves a community of 1500 people.

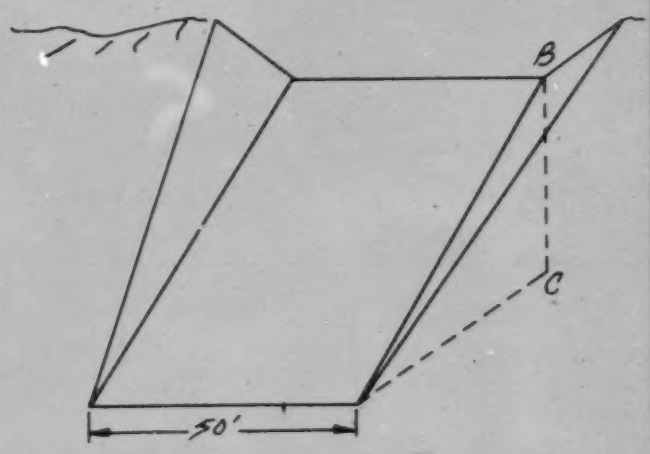
Problem: Calculate volume of water in tank, and assume per capita consumption to be 110 gal. per day, how long will supply last, or how many times must it be pumped full in 24 hours?

B. Problems without solutions.

5. When laying out a ℄ for a R/W, an obstacle, consisting of trees and a marsh, was encountered by the survey crew. In order to get around the obstacle they had to deflect to the right N34°32'E. from a bearing of true North. They chained out a distance of 300 ft. At this point they were in a position to return to the ℄. With respect to their present bearing what angle must they turn in order to intersect their ℄ at a right angle? How far must they travel to reach the ℄ ?

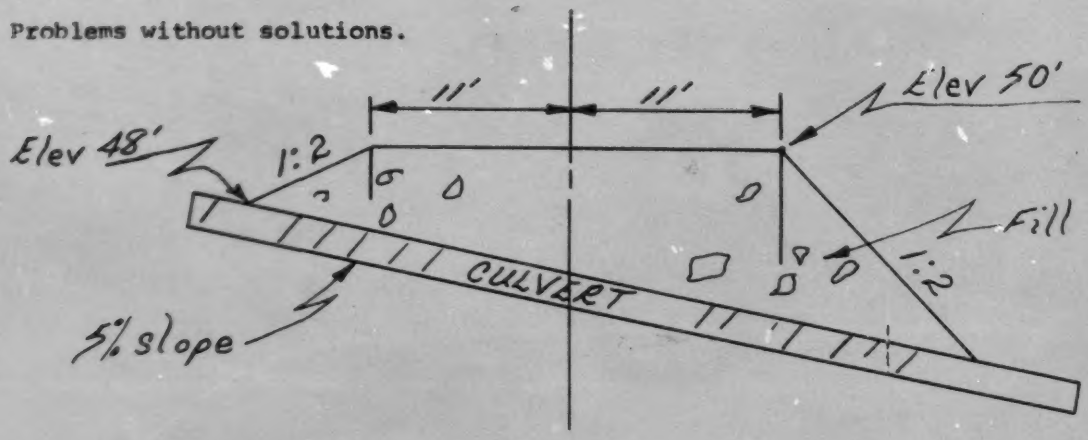
6. In bidding a job one must calculate both cut and fills. Below is a situation in which a highway is to pass over a small hill. The approach side has a slope of 27% and is 400' in length. The descent side is 350' in length. The highway is to have an approach of 5% slope. The banks are to have a 1:1 slope. What is the total length of the road bed? How many cubic yds. must be removed from the approach side? What is the % of slope for the descent road bed?





B. Problems without solutions.

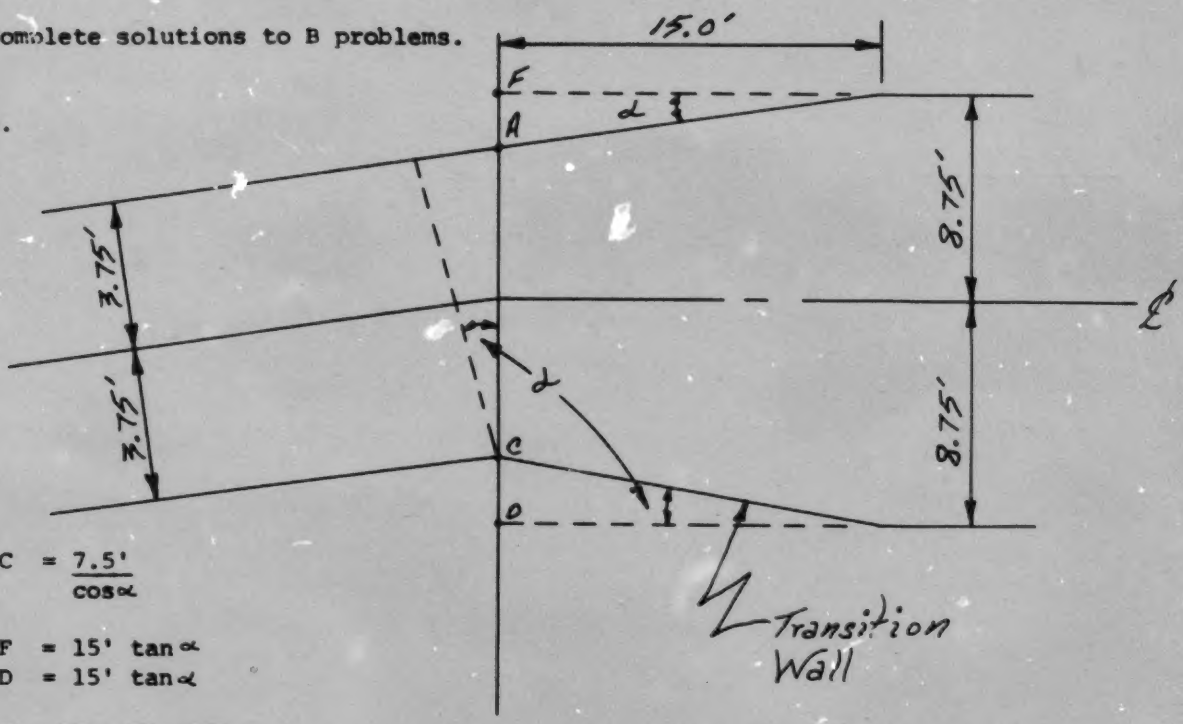
7.



Problem: The culvert extends 2' beyond each bank. One needs to install the culvert before fill is put in place. How long should the culvert be?

Complete solutions to B problems.

3.



$AC = \frac{7.5'}{\cos\alpha}$

$AF = 15' \tan\alpha$

$CD = 15' \tan\alpha$

$AC + AF + CD = FD$

$\frac{7.5'}{\cos\alpha} + 15' \tan\alpha + 15' \tan\alpha = 17.5'$

$\frac{7.5'}{\cos\alpha} + 30' \tan\alpha = 17.5'$

$\frac{7.5'}{\cos\alpha} + \frac{30' \sin\alpha}{\cos\alpha} = 17.5'$

$\frac{7.5' + 30' \sin\alpha}{\cos\alpha} = 17.5'$

$7.5' + 30' \sin\alpha = 17.5' \cos$

$17.5' \cos\alpha - 30' \sin\alpha - 7.5' = 0$

Substituting and cancelling units

$17.5 \cos\alpha - 30\sqrt{1 - \cos^2\alpha} = 7.5$

$17.5 \cos\alpha - 7.5 = 30\sqrt{1 - \cos^2\alpha}$

$306.25 \cos^2\alpha - 261.50 \cos\alpha + 56.25 = (900)(1 - \cos^2\alpha)$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin\theta = \sqrt{1 - \cos^2\theta}$

Complete solutions to B problems.

3. continued.

$$306.25 \cos^2\alpha - 262.50 \cos\alpha + 56.25 = 900 - 900 \cos^2\alpha$$

$$1206.25 \cos^2\alpha - 262.50 \cos\alpha - 843.75 = 0$$

$$\cos\alpha = \frac{262.50 \pm \sqrt{68906.25 + 4071093.75}}{2412.50}$$

Discarding the negative value

$$\cos\alpha = \frac{262.50 + \sqrt{4140,000}}{2412.50}$$

$$= \frac{262.5 + 2034.70}{2412.50}$$

$$= \frac{2297.2}{2412.50}$$

$$= .95221$$

$$= 17.79^\circ$$

$$= 17^\circ\ 47'\ 8"$$

Level II

4. To find the volume of the tank, split it into 2 sections. The first is a hemisphere 18' in diameter, the second a cylinder with the same diameter.

The volume of a cylinder $= \pi r^2 h$
$= (3.14)(9')^2 (27.42' - 9')$
$= (3.14)(9')^2 (18.42')$
$= 4684.1$ cu ft

The volume of a hemisphere
$= 1/2 \left[4/3\ r^3 \pi\right]$
$= 1/2 \left[4/3\ \pi (9')^3\right]$
$= 1/2 \left[(4/3)(3.14)(729 \text{ cu ft})\right]$
$= 1/2 \left[3052.08 \text{ cu ft}\right]$
$= 1526.04$ cu ft

Therefore, the total volume of the tank is

$V_T = 4684.1$ cu ft + 1526.04 cu ft

$= 6210.13$ cu ft

Complete solutions to B problems.

4. continued.

Converting

$$(6210.13\ \text{cu ft})\left(7.48052\ \frac{\text{gal}}{\text{cu ft}}\right) = 46{,}455 \text{ gallons}$$

Since the consumption is 110 gal per person per day, they use

(110 gal/person) (1500 people) = 165,000 gallon

So, the tank must be filled

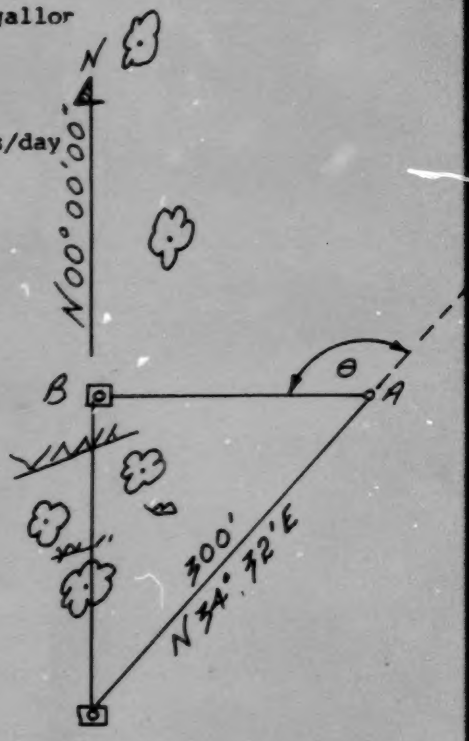
$$\frac{165{,}000\ \text{gallons/day}}{46{,}445\ \text{gallons}} = 3.55 \text{ times/day}$$

5. We want to find angle θ and the length of segment $\overline{AB}$.

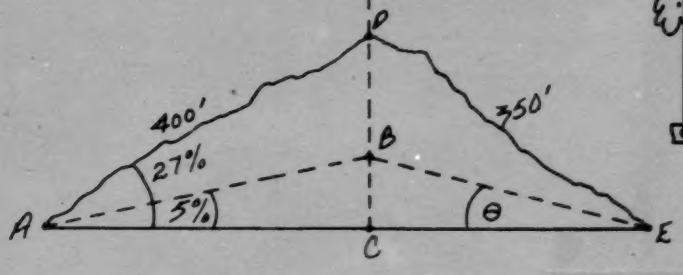
The angle $= 90^\circ + 34^\circ 32'$
$= 124^\circ 32'$

So, with respect to their present bearing they must turn left $124^\circ 32'$.
The length of seqment AB is

300' sin $34^\circ 32'$ = 169.14'



6.



To find the length of the road bed, find the combined lengths of segment $\overline{AB}$ and $\overline{BE}$. To find the length AB, first find length AC.

Ac = 400' cos 27%
= 400' cos 15.11°
=(400') (.97)
Then = 386.17 ft
AB = AC / cos 5%
= 386.17' / cos 5%
= 386.17'/ cos 2.86°
= 386.17'/ .9988
= 386.65 ft

6. continued.

Now, to find the length of BE, first find the length of DC.

$DC = 400' \sin 27\%$
$= 400' \sin 15.11^{\circ}$
$= (400')\ (.26)$
$= 104.27 \text{ ft}$

Then, using pythagorean theorem

$CE^2 = (350')^2 - DC^2$
$CE^2 = 122500 \text{ sq ft} - 10871.47 \text{ sq ft}$
$CE^2 = 111,628,53 \text{ sq ft}$
$CE = 334.11 \text{ ft}$

$BC = AC \tan 5\%$
$= 386.17' \tan 2.86^{\circ}$
$= (386.17')(.05)$
$= 19.31'$

Again, using pythagorean theorem

$BE^2 = BC^2 + CE^2$
$= (19.31)^2 + (334.11')^2$

$= 372.81 \text{ sq ft} + 111,628.53 \text{ sq ft}$

$= 112,001.35 \text{ sq ft}$

$BE = 334.67 \text{ ft}$

Therefore the total road bead length is

$334.67 \text{ ft} + 386.65 \text{ ft} = 721.32 \text{ ft}$

The angle between $\overline{EC}$ and $\overline{EB}$ is the slope of the descent road.

$$\tan \theta = \frac{BC}{CE}$$
$$= \frac{19.31'}{334.11}$$

$$\tan \theta = .0578$$

The slope of the descent must equal 5.78%.

Complete solutions to B problems.

6. continued.

Finally, solve for the amount of rock to be removed from the approach side. The first step is to solve for the area of the wedge that excludes the banks. This involves finding the area of the entire wedge ACD and subtracting wedge ACB.

Volume ACD = (AC/2)(CD) (50')
= (386.17'/2)(104.27')(50')
= 1,006,617.6 cu ft.

Volume ACB = (AC/2)(CB)(50')
= (386.17'/2) (19.13')(50')
= 186,410.66 cu ft

Volume ABD = Volume ACD - Volume ACB
= 1,006,617.6 cu ft - 186410.66 cu ft
= (820,206.94 ~~cu ft~~)(1 cu yd/ 27 ~~cu ft~~)
= 30,378.03 cu yds

Now, solve for the volume of the pyramidal sections of the bank. The volume of one is 1/3 bh. The base of the pyramid is a triangle, so find the area. The slope is 1:1. The difference in height is

DC - BC
= 104.27 ft - 19.31 ft
= 84.96 ft

So the triangle area is 1/2 bh

=(84.96 ft /2) (84.96 ft)

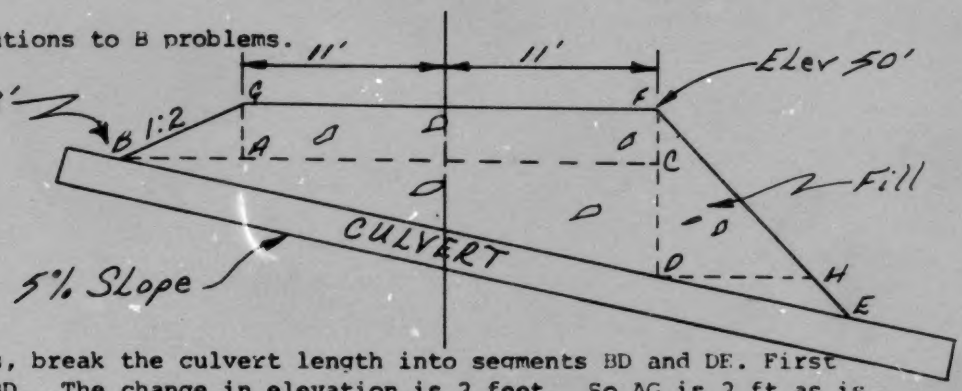
= 3609.1 sq ft

The volume of the pyramids is 1/3 bh

= 1/3 (3609.1 sq ft)(AC)
= 1/3 (3609.1 sq ft)(386.17 ')
= 364,575.38 cu ft
= 17,206.50 cu yds

So, a total of 17,2065cu yds + 30,378.03 cu yds = 47,584.53 cu yds must be removed from the approach side..

Complete solutions to B problems.



To solve this, break the culvert length into segments BD and DE. First find length BD. The change in elevation is 2 feet. So AG is 2 ft as is CF, and AB is 4 ft. So length BC is

$$4 \text{ ft} + 22 \text{ ft} = 26 \text{ ft}$$

Since the slope of the culvert is 5%, length BD equals

$$\begin{aligned} &BC/\cos 5\% \\ &= 26 \text{ ft}/\cos 2.86^\circ \\ &= 26.03 \text{ ft} \end{aligned}$$

Now, to find DE, first we must find DF.

$$\begin{aligned} DF &= CD + CF \\ &= BC \tan 5\% + 2 \text{ ft} \\ &= 26' \tan 5\% + 2 \text{ ft} \\ &= 1.30 + 2 \text{ ft} \\ &= 3.30 \text{ ft} \end{aligned}$$

We also need to know angle EFD. In triangle FDH,

$$1/2 = \frac{FD}{DH}$$

$$2FD = DH$$

$$(2)(3.30\text{ft}) = DH$$

$$6.60 \text{ ft} = DH$$

$$\tan \angle DFH = \tan \angle DFE = 6.60 \text{ ft}/3.30 \text{ ft} = 2$$

$$\angle DFH = \angle DFE = 63.43^\circ$$

Therefore, since the sum of the interior angles if a triangle is 180°, angle DEF $= 180^\circ - (92.86^\circ + 63.43^\circ)$

$$= 180^\circ - (156.29^\circ)$$

$$= 23.71^\circ$$

From the Law of Sines

$$\frac{DF}{\sin \angle DEF} = \frac{DE}{\sin \angle DFE}$$

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Complete solutions to B problems.

7. continued.

$$\frac{3.30 \text{ ft}}{.40} = \frac{DE}{.89}$$

$$DE = \frac{(330 \text{ ft})(.89)}{.40}$$

$$= 7.38'$$

So the culvert should be

$$2' + 26.03' + 7.38' + 2' = \underline{37.41 \text{ ft}}$$

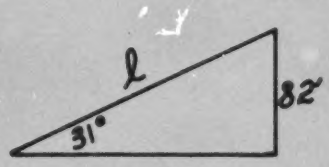
A. Problems with Solutions

1. The most efficient operating angle of elevation for a conveyor used to elevate sawdust to a digester is 31°. If the sawdust is to be elevated 82', what length of conveyor is needed?

Solution:

L = length of conveyor

$\sin 31^\circ = \frac{82 \text{ ft}}{L}$



$L = \frac{82 \text{ ft}}{\sin 31} = \frac{82 \text{ ft}}{0.515} = 160 \text{ ft}$

2. What is the length of set, if the offset is 22 1/2° and the travel is 32 1/8"?

Solution:

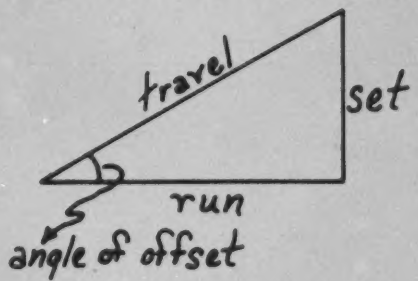
In pipe fitting; set, run, travel and angle of offset are the parts of a right triangle as labeled below.

$\sin 22\ 1/2^\circ = \frac{\text{set}}{\text{travel}}$

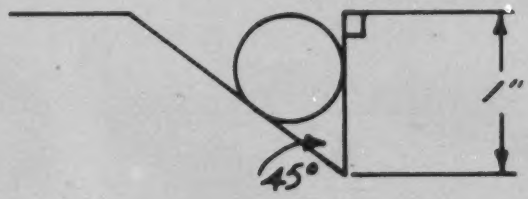
$= \frac{\text{set}}{32\ 1/8"}$

$0.3827 = \frac{\text{set}}{32\ 1/8"}$

set = (32.125 in)(0.3827)
= 12.294 in or 1.024 ft

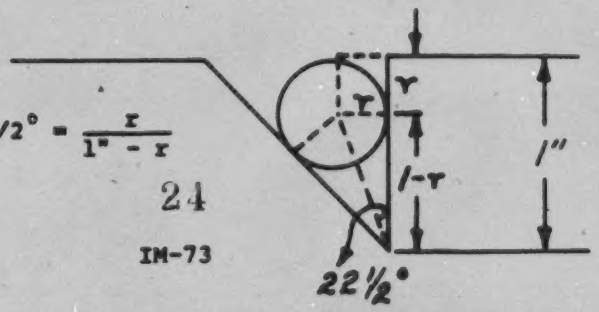


3. To accurately measure a cut, as shown, the size of a plug which will be flush with the top surface is calculated. The plug is turned to size and then used to calculate the accuracy of the cut. Find the size of the plug to use in the cut illustrated.



Solution:

$\tan 22\ 1/2^\circ = \frac{r}{1" - r}$



A. Problems with Solutions (continued)

3. Solution: (continued)

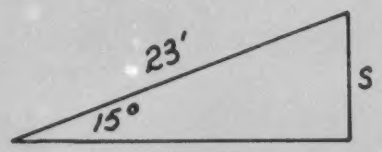
$$0.4142 = \frac{r}{1" - r}$$

$$(0.4142)(1" - r) = r$$
$$0.4142" - 0.4142r = r$$
$$0.4142" = 1.4142r$$

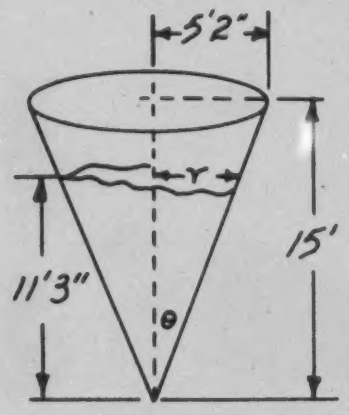
$$r = \frac{0.4142"}{1.4142} = 0.2929 \text{ in} \qquad \text{diameter} = 2r = 2(0.2929 \text{ in}) = 0.5858 \text{ in}$$

B. Problems without Solutions

4. A millwright is to weld supports for a 23' conveyor so that it will operate at a 15° angle. What is the length of the supports?

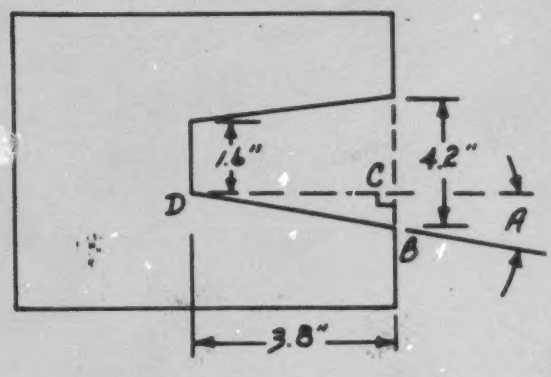


5. A conical tank with point down of height 15' and radius 5' 2" is filled to a height of 11' 3" with liquid. How many gallons of liquid are in the tank?

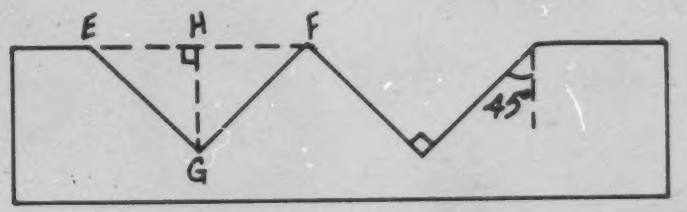


B. Problems without Solutions (continued)

6. A casting is to be machined internally as shown. In order to accomplish this, the measurement of angle A is required. Calculate this to the nearest degree.

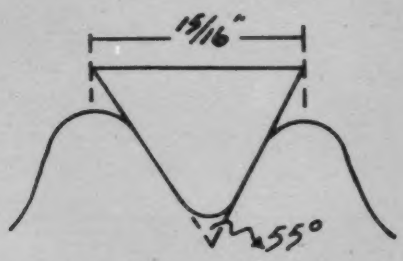


7. The template for the hydraulic copying attachment on a shaper is as illustrated. If the depth of cut HG is 1.75", what is the measurement of EF? (EG = GF)

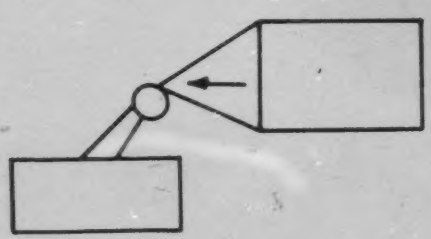


B. Problems without Solutions (continued)

8. In a certain type of chain sprocket the sides of the teeth make an angle of 55° with each other. The depth of the teeth is 2/3 of the depth of the triangle formed by extending the sides. What is the depth of the teeth on a sprocket with a pitch of 15/16"?



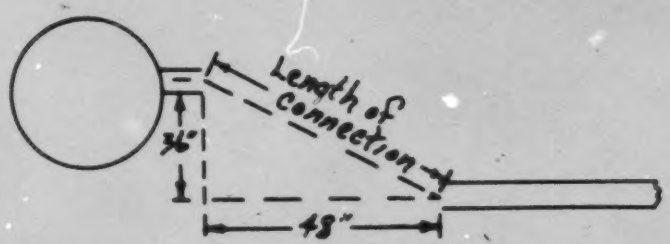
9. A certain machine has a limit switch like the one in the drawing. In order to function properly the pointed rod must force the top of the switch to move 0.75" vertically while it moves 1.37" horizontally. What must the angle at the tip of the rod be in order to achieve proper functioning of the switch?



B. Problems without Solutions (continued)

10. What is the length of run and travel of a 45° offset with a set of 15"?

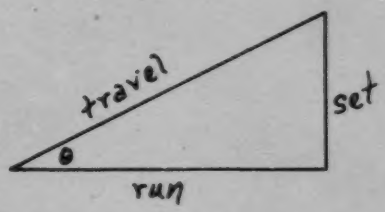
11. A pipe fitter is to connect a pipe line to a tank. The run from the pipe line to the tank is 48" while the set is 36". Will the pipe fitter be able to use standard fittings? (pipe connections of 22 1/2°, 30°, 45°, 60° or 90°)



B. Problems without Solutions (continued)

12. A welder is asked to make three offsets. The three cases are given below; in each case find the angle of the offset.

(a) set = 18" run = 24"
(b) travel = 36" run = 17"
(c) set = 15 3/4" travel = 21 1/2"



13. A rail is to be constructed around the top of a cylindrical tank with a diameter of 14'. If the rail is to be constructed in 15 sections, how long should each section be?

14. Twelve bolt holes are to be drilled in a cylinder head on the circumference of a 14" circle. What is the straight line distance between the centers of the holes?

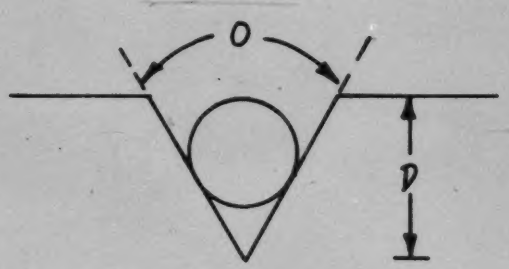
B. Problems without Solutions (continued)

15. Nine holes are to be cut in a circular piece of flat metal on the circumference of a circle 7 1/4" in diameter. What should be the distance between the centers of the holes?

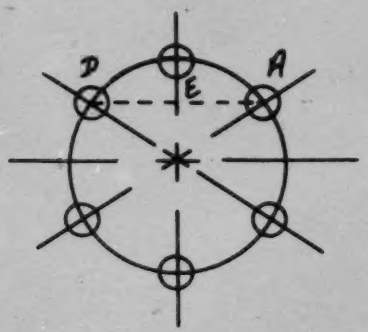
16. Five holes are to be equally spaced around a circle. The holes are to be 4.70" apart, center to center. Calculate the diameter of the circle they will lie on.

B. Problems without Solutions (continued)

17. Find the size of plug needed to measure the cut illustrated if D = 3 1/8" and $\theta = 45^{\circ}$.



18. In order to check the accuracy of our work after boring the holes, we can measure across alternate holes AD in the figure. Find what the measurement of AD should be if the holes are bored on a 6" circle. The holes are equally spaced.



Complete Solutions to B Problems

4. S = length of supports

$$\sin 15^\circ = \frac{S}{23 \text{ ft}}$$

$$S = (23 \text{ ft})(\sin 15^\circ) = (23 \text{ ft})(0.2588) = 6 \text{ ft}$$

5. We must find r

$$\tan\theta = \frac{5' \ 2"}{15'} = \frac{5.167 \text{ ft}}{15 \text{ ft}} = 0.3445$$

$$\tan\theta = \frac{r}{11' \ 3"} = \frac{r}{11.25 \text{ ft}}$$

$$0.3445 = \frac{r}{11.25 \text{ ft}}$$

$$r = (0.3445)(11.25 \text{ ft}) = 3.876 \text{ ft}$$

$$V = 1/3 \pi r^2 h = (1/3)(3.14)(3.876 \text{ ft})^2 (11.25 \text{ ft})$$

$$= 177 \text{ cu ft}$$

$$= (177 \text{ cu ft})(7.48 \text{ gal/cu ft}) = 1{,}323 \text{ gal}$$

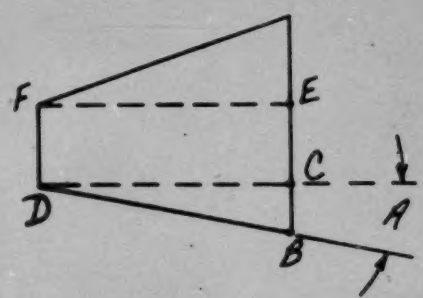
6. $$CB = \frac{4.2" - 1.6"}{2} = 1.3 \text{ in}$$

DC = 3.8 in

$$\tan A = \frac{CB}{DC} = \frac{1.3 \text{ in}}{3.8 \text{ in}}$$

$$= 0.3421$$

$$A = 18.9^\circ$$

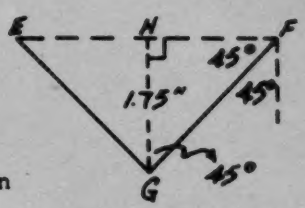


7. $$\tan 45^\circ = \frac{HF}{1.75"}$$

$$1 = \frac{HF}{1.75"}$$

HF = 1.75"

EF = 2(1.75") = 3.5 in



Alternate solution: HFG is isosceles, so HF = 1.75" and again EF = 2(HF) = 3.5 in

Complete Solutions to B Problems (continued)

8. $BC = (1/2)(15/16") = 15/32$ in

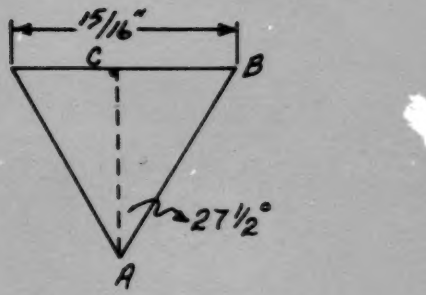
$CAB = (1/2)(55^\circ) = 27.5^\circ$

$\tan 27.5^\circ = \frac{BC}{AC}$

$0.5206 = \frac{0.4688 \text{ in}}{AC}$

$AC = \frac{0.4688 \text{ in}}{0.5206} = 0.9004$ in

Depth = (2/3)(0.9004 in) = 0.6003 in

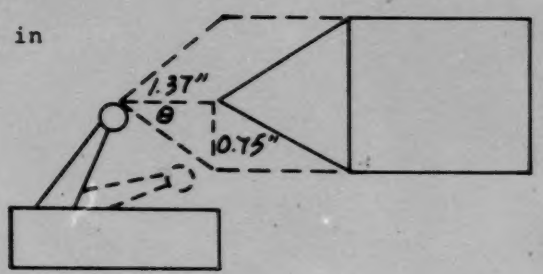


9. θ = 1/2 of angle desired

$\tan\theta = \frac{0.75 \text{ in}}{1.37 \text{ in}} = 0.5474$

$\theta = 28.7^\circ$

Angle at end of switch = $2(28.7^\circ) = 57.4^\circ$

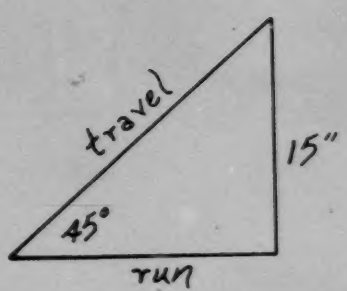


10. run = 15 in

$\sin 45^\circ = \frac{15 \text{ in}}{\text{travel}}$

$0.7071 = \frac{15 \text{ in}}{\text{travel}}$

travel = $\frac{15 \text{ in}}{0.7071} = 21.21$ in $\approx$ 21 1/4 in

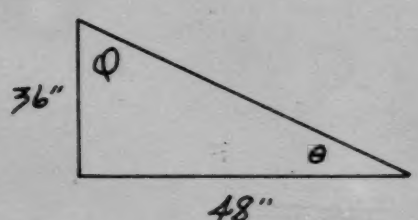


11. $\tan\theta = \frac{36 \text{ in}}{48 \text{ in}} = 0.75$

$\theta = 36.9$

$\tan\theta = \frac{48 \text{ in}}{36 \text{ in}} = 1.3333$

$\theta = 53.1^\circ$



Neither 36.9° nor 53.1° can be made from a standard pipe fitting.

Complete Solutions to B Problems (continued)

12. (a) $\tan\theta = \frac{\text{set}}{\text{run}} = \frac{18 \text{ in}}{24 \text{ in}} = 0.75$

$\theta = 36.9^\circ$

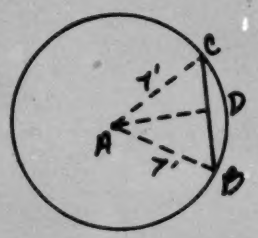
(b) $\cos\theta = \frac{\text{run}}{\text{travel}} = \frac{17 \text{ in}}{36 \text{ in}} = 0.4722$

$\theta = 61.8^\circ$

(c) $\sin\theta = \frac{\text{set}}{\text{travel}} = \frac{15\ 3/4 \text{ in}}{21\ 1/2 \text{ in}} = \frac{15.75}{21.5} = 0.7326$

$\theta = 47.1^\circ$

13. $CAB = \frac{360^\circ}{15} = 24^\circ$



$CAD = (1/2)(24^\circ) = 12^\circ$

$\sin 12^\circ = \frac{CD}{7 \text{ ft}}$

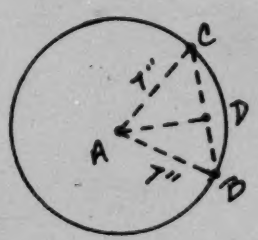
$0.2079 = \frac{CD}{7 \text{ ft}}$

$CD = (0.2079)(7 \text{ ft}) = 1.45 \text{ ft}$

CB = length of section = $2CD = (2)(1.45 \text{ ft}) = 2.9 \text{ ft}$

14. $CAB = \frac{360}{12} = 30^\circ$

$CAD = (1/2)(30^\circ) = 15^\circ$



$\sin 15^\circ = \frac{CD}{7 \text{ in}}$

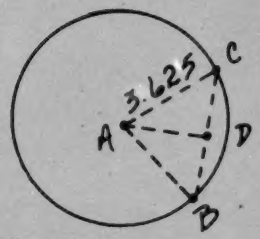
$0.2588 = \frac{CD}{7 \text{ in}}$

$CD = (0.2588)(7 \text{ in}) = 1.8 \text{ in}$

CB = distance between centers
= $2\ CD = 3.6 \text{ in}$

Complete Solutions to B Problems (continued)

15.



$CAB = \frac{360^\circ}{9} = 40^\circ$

$CAD = (1/2)(40^\circ) = 20^\circ$

$AC = (1/2)(7.25 \text{ in}) = 3.625 \text{ in}$

$\sin 20^\circ = \frac{CD}{3.625 \text{ in}}$

$0.3420 = \frac{CD}{3.625 \text{ in}}$

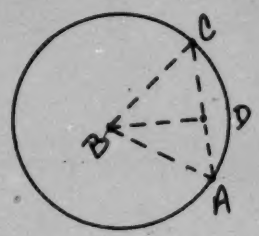
$CD = (0.3420)(3.625 \text{ in}) = 1.24 \text{ in}$

CB = distance between centers = $2CD = 2.48$ in

16.

$AC = 4.70$ in

$CD = (1/2)(4.70 \text{ in}) = 2.35$ in



$CBA = \frac{360^\circ}{5} = 72^\circ$

$CBD = (1/2)(72^\circ) = 36^\circ$

$\sin 36^\circ = \frac{CD}{CB}$

$0.5878 = \frac{2.35 \text{ in}}{CB}$

$CB = \frac{2.35 \text{ in}}{0.5878} = 4.00$ in

Level 3

17.

$\sin 22\ 1/2^\circ = \frac{r}{3.125" - r}$

$0.3827 = \frac{r}{3.125" - r}$

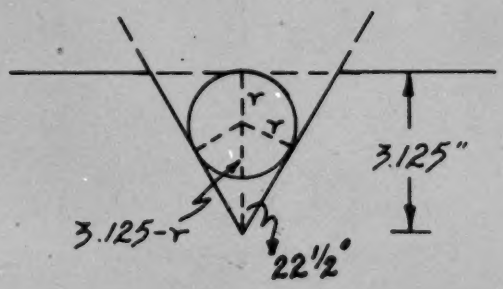
$(0.3827)(3.125" - r) = r$

$1.196" - 0.3827r = r$

$1.196 \text{ in} = 1.3827r$

$r = 0.8650$ in

diameter = 1.730 in



Complete Solutions to B Problems (continued)

18. $\angle DCE = \frac{360^\circ}{6} = 60^\circ$

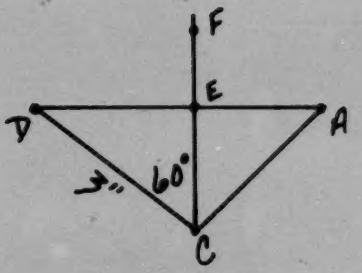
$DC = (1/2)(6 \text{ in}) = 3 \text{ in}$

$\sin 60^\circ = \frac{DE}{DC} = \frac{DE}{3 \text{ in}}$

$0.8660 = \frac{DE}{3 \text{ in}}$

$DE = 2.600 \text{ in}$

$AD = 2DE = 5.200 \text{ in}$



A. Problems with Solutions

1. The torque on a 1/2" head bolt is 80 ft lb. If the thread pitch is 1/16", what force is the bolt exerting on the head? (Steel to steel coefficient of friction is 0.8.)

   Solution:

   $$\text{Force} = (2\pi)(12\ \cancel{in}/\cancel{ft})(16/\cancel{in})(80\ \cancel{ft}\ \text{lbs})(0.2)(\cos 30^\circ)$$

   $$= (6.2832)(12)(16)(80\ \text{lbs})(0.2)(0.866)$$

   $$= 16{,}715.52\ \text{lbs}$$

2. Assuming that one bolt is 3/8" diameter with a 20 thread and another bolt is 3/8" diameter with a 32 thread, what would be the difference in force if each were torqued to 60 ft lbs? (Coefficient of friction = 0.8.)

   Solution:

   Force on 32 thread:

   $$(32/\cancel{in})(12\ \cancel{in}/\cancel{ft})(2)(3.1416)(\cos 30^\circ)(0.2)(60\ \cancel{ft}\ \text{lb}) =$$

   $$(32)(12)(6.2832)(0.866)(0.2)(60\ \text{lbs}) = 25{,}073.28\ \text{lbs}$$

   Force on 20 thread:

   $$(20/\cancel{in})(12\ \cancel{in}/\cancel{ft})(2)(3.1416)(\cos 30^\circ)(0.2)(60\ \cancel{ft}\ \text{lbs}) =$$

   $$(20)(12)(6.2832)(0.866)(0.2)(60\ \text{lbs}) = 15{,}670.80\ \text{lbs}$$

   $$\text{Difference} = 25{,}073.28\ \text{lbs} - 15{,}670.80\ \text{lbs}$$

   $$= 9{,}402.48\ \text{lbs of force}$$

3. A 1/2" diameter engine head bolt with 20 threads per inch is torqued to 70 ft lbs. What is the tension per square inch of cross sectional area on the bolt? (Coefficient of friction = 0.8)

   Solution:

   $$(2\pi)(12\ \cancel{in}/\cancel{ft})(20/\cancel{in})(\cos 30^\circ)(0.2)(70\ \cancel{ft}\ \text{lbs}) =$$

   $$(6.2832)(12)(20)(0.866)(0.2)(70\ \text{lbs}) = 18{,}282.6\ \text{lbs}$$

   Each bolt has a tension of 18,282.6 lbs and the cross-sectional area of the bolt is $\left[(1/2")^2(0.7854)\right] = 0.19635$ sq in

   $$\text{Tensile stress} = \frac{18{,}282.6\ \text{lbs}}{0.19635\ \text{sq in}} = 93{,}112.3\ \text{PSI}$$

B. Problems without Solutions

4. A Chevrolet 350 engine has 17 head bolts which should be torqued to 70 ft lbs. The bolts are 1/2" diameter and have a 20 thread. What force does each bolt exert on the head? What total force holds the head and the block together? (Coefficient of friction = 0.8.)

5. If the pitch diameters of a set of timing gears are 2.875" and 5.75" respectively and their centers are 6.005" apart, what arc does the chain subtend on each gear?

6. A 1956 Ford had a change of direction of 7° maximum at the front universal joint and used a Crooks universal joint. If the RPM of the transmission output shaft were 2400, what would be the maximum and minimum RPM of the drive shaft? (Crooks universal joint: the universal joint making use of cross and bearings.)

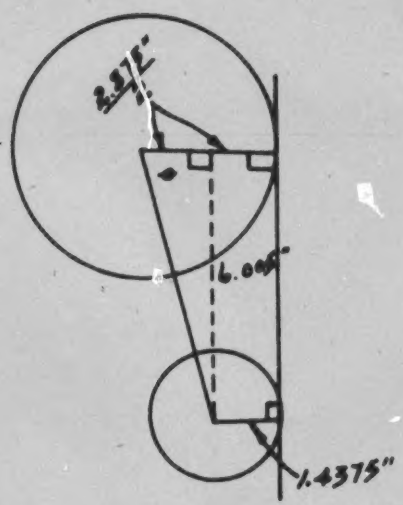
Complete Solutions to B Problems

4. Force per bolt = $(2\pi)(20/\text{in})(12\ \text{in/ft})(\cos 30^\circ)(0.2)(70\ \text{ft lbs})$
$= (6.2832)(20)(12)(0.866)(0.2)(70\ \text{lbs})$
$= 18{,}282.6\ \text{lbs}$

Total force = $(18{,}282.6\ \text{lbs/bolt})(17\ \text{bolts})$

$= 310{,}804.2\ \text{lbs}$

5.



$\cos\theta = \frac{1.4375"}{6.005"} = 0.23938$

$\theta = 76^\circ\ 9'$

$2\theta = 152^\circ\ 18'$ arc on crankshaft gear

$360^\circ - 152^\circ\ 18' = 207^\circ\ 42'$ arc on cam gear

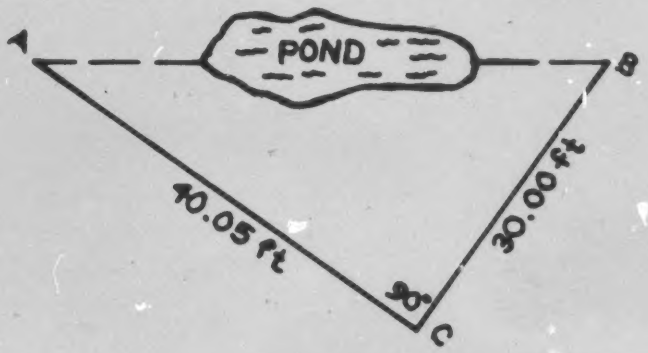
6. While the output shaft would make $(\pi d)(2400\ \text{RPM})$, the drive shaft would make $(\cos 7^\circ \pi)(2400\ \text{RPM})$ and must turn slower to make the shorter distances at 90° of the turn the process would reverse.

Lowest RPM = $(\cos 7^\circ)(2{,}400\ \text{RPM}) = (0.9926)(2{,}400\ \text{RPM}) = 2{,}382.24\ \text{RPM}$

Highest RPM = $(\sec 7^\circ)(2{,}400\ \text{RPM}) = (1.00751)(2{,}400\ \text{RPM}) + 2{,}418.02\ \text{RPM}$

A. Problems with Solutions

1. Compute the distance between points A and B using the three measurements given in the sketch. Obtain a "rough check" of your answer through your knowledge of a 3-4-5 triangle.



Solution:

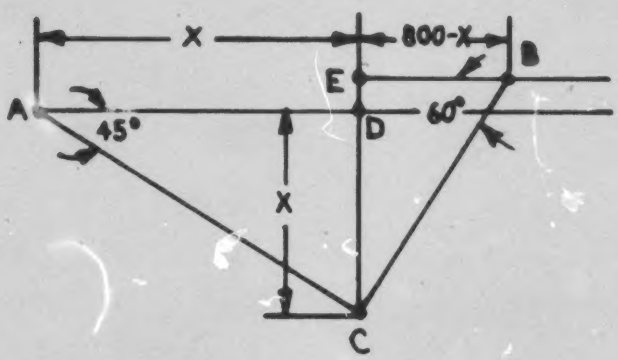
$$\tan B = \frac{40.05 \text{ ft}}{30.00 \text{ ft}} = 1.335$$

$$B = \tan^{-1} 1.335 = 53.16^\circ$$

$$\sin B = \frac{40.05 \text{ ft}}{AB}$$

$$AB = \frac{40.05 \text{ ft}}{\sin B} = \frac{40.05 \text{ ft}}{\sin 53.16^\circ} = \frac{40.05 \text{ ft}}{0.8004} = 50.04 \text{ ft}$$

2. Points A and B are 800 ft apart horizontally and B is 150 ft higher than A. Point C lies in a valley between them. A transit line from A to C dips 45° from the horizontal and from B to C dips 60°. How far is C below A?



Solution:

A. Problems with Solutions (continued)

2. Solution: (continued)

Refer to the sketch. D is directly above C. Since angle CAD and ACD are 45°, their opposite sides are equal. This unknown value is called X and represents the depth of C below A. Point E is located directly above C and at the same elevation as B.

Applying the Sine Law to triangle BEC,

$$\frac{\sin 60^\circ}{150 \text{ ft} + X} = \frac{\sin 30^\circ}{800 \text{ ft} - X}$$

$$\frac{0.8660}{150 \text{ ft} + X} = \frac{0.5000}{800 \text{ ft} - X}$$

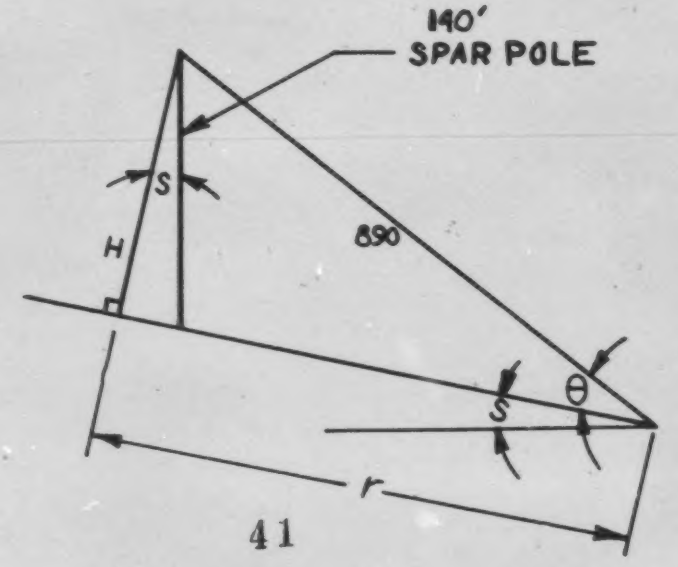
$$(0.8660)(800 \text{ ft} - X) = (0.5000)(150 \text{ ft} + X)$$
$$692.8 \text{ ft} - 0.8660X = 75.0 \text{ ft} + 0.5X$$
$$617.8 \text{ ft} = 1.366X$$

$$\frac{617.8 \text{ ft}}{1.366} = X$$

$$452.3 \text{ ft} = X$$

3. Determine the area in acres which can be logged around a 140 foot spar pole standing in an area where the slope is a uniform 66%. The length of line available (from the top of the spar pole) is 890 ft.

Solution:



A. Problems with Solutions (continued)

3. Solution: (continued)

See sketch. The area is circular with a radius r. h is the perpendicular distance from the top of the spar to the ground. S represents the slope angle of 66% and represents the minimum angle between the line and the ground.

$h = (140 \text{ ft}) \cos(\tan^{-1} 0.66)$
$= (140 \text{ ft})(\cos 33.42^\circ)$
$= (140 \text{ ft})(0.8346)$
$= 116.8 \text{ ft}$

$r = (890 \text{ ft})(\cos \theta)$
$= (890 \text{ ft}) \cos\left(\sin^{-1} \frac{116.8}{890}\right)$

$= (890 \text{ ft}) \cos(\sin^{-1} 0.1312)$
$= (890 \text{ ft})(\cos 7.5410^\circ)$
$= (890 \text{ ft})(0.9914)$
$= 882.3 \text{ ft}$

$$\text{Area} = \frac{\pi r^2}{43{,}560 \text{ sq ft/acre}}$$

$$= \frac{(\pi)(882.3 \text{ ft})^2}{43{,}560 \text{ sq ft/acre}}$$

$$= \frac{(3.1416)(778{,}453.29 \text{ sq ft})}{43{,}560 \text{ sq ft/acre}}$$

$= 56 \text{ acres}$

B. Problems without Solutions

4. Between stations 13 + 50 and 27 + 30 a road has a uniform grade of 5 degrees. a) What is the % grade of this portion of the road? b) What is the difference in elevation between the two stations?

Note: Station 13 + 50 denotes a point which is a horizontal distance of 1,350 ft from a reference point (station) having the designation 00 + 00.
Difference in elevation, D.E., equals the horizontal distance between the points multiplied by the Tan 5°.

B. Problems without Solutions (continued)

5. If the declination on a compass was set at 20° 30'E instead of the correct value of 19° 00' E how much line error would accumulate in a distance of 1/2 mile? Would the error be to the right or left?

6. A man measures off six lengths with a 100 ft steel tape along the surface of a road. He later determines that the length of the tape was 100.04 feet and that the road had a uniform slope of 5%. Write an expression for the actual horizontal distance between the two end points and then solve it, rounding to the nearest one tenth of a foot.

7. A forester runs a line one half mile in length from point A to point B and finds himself 34.5 ft north of point B due to the declination set off on his compass. Point B is known to be due east of point A. a) If the declination set off on his compass was 21° 30' E, what declination should he have used? b) Assuming that the declination could not be changed, what bearing should he use in retracting the line from point B to point A?

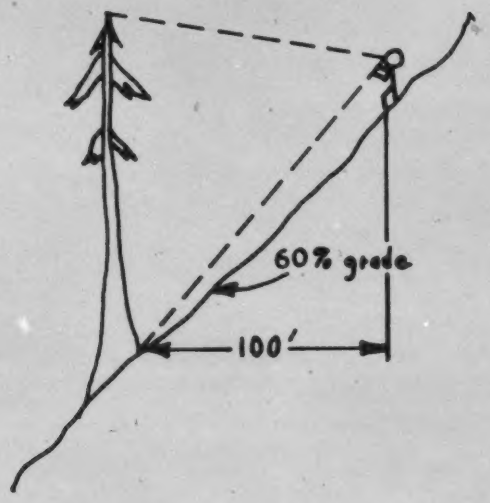
B. Problems without Solutions (continued)

8. In the process of surveying a tract of timber it becomes necessary to determine the distance to an inaccessable but visable point, C. A distance between two points A and B of 500.0 feet is measured. Then angles ABC and BAC are measured and found to be 50 15' and 65 18' respectively. Calculate the perpendicular distance from C to the line AB, also the distance AC.

9. In measuring a line with a 100 foot tape the zero end is held 2 feet 6 inches too high: What is the error in the tape measurement? Round your answer to two decimal places.

B. Problems without Solutions (continued)

10. A man uses a percent abney to determine the height of a tree. The base of the tree is below him and the average slope of the ground between himself and the tree is 60%. For ease of calculations he desires to have the horizontal distance between himself and the tree equal to 100 feet. Determine the slope distance from the man's feet to the base of the tree. After measuring off this distance the % readings taken for the top and base of the tree are + 15 and - 140 , respectively. Determine the height of the tree.



B. Problems without Solutions (continued)

11. Given the following slope distances and abney readings determine the horizontal distances to an accuracy of 0.01 ft.

Slope Distance	Abney	Horizontal Distance
100.04 ft	31%	_____
197.13 ft	27%	_____
85.79 ft	59%	_____

12. Between stations 5 + 20 and 9 + 50 a road has a uniform grade of 15%. Determine the difference in elevation, D.E. and the slope distance, S.D. between the two points.

    Note: Station 5 + 20 indicated a point which is a horizontal distance of 520 feet from a reference station designated 00 + 00.

13. A distance of 200 ft is measured along a 12% slope. a) What is the horizontal distance? b) What is the verticle distance between the two points? Note: Round your answer to the nearest tenth of a foot.

B. Problems without Solutions (continued)

14. What slope distance would be measured with a 100 ft tape (which is 0.05 ft too short) if the slope is 32% and the true horizontal distance is to be 937.5 ft? Note: Round your answer to the nearest tenth of a foot.

15. The distance along the slope is 235 ft. The horizontal distance is 224 ft. Determine: a) The % slope, and b) the difference in elevation, DE.

16. A random line is a convenient device to use in transit and tape surveying when a line must be run between two points which are not in line of sight. AB is such a line. Run AC to the right of AB until an unobstructed view of B from C is available. Measurements are as follows: AC length is 5,102.5 ft. AC bearing N 89° 10' E, CB length is 178.6 ft. Angle ACB is 93° 16'. Determine the length and bearing of AB.

B. Problems without Solutions (continued)

17. The difference in elevation between two stations on a traverse is - 47.4 ft and the abney reading with a percent abney is - 27.5 percent. Determine the slope distance between the stations in chains and links. Note: 1 chain = 100 links = 66 ft.

4. a) Grade = $\tan 5^\circ = 0.0875 = 8.75\%$

b) D.E. = (H.D.)($\tan 5^\circ$)
H.D. = 2,730 ft - 1,350 ft = 1,380 ft
D.E. = (1,380 ft)(0.0875)
= 120.7 ft

5. Error = (5,280 ft/~~mile~~)(1/2 ~~mile~~)($\sin 1.5^\circ$)
= (2,640 ft)(0.0262)
= 69 ft

The error is to the left.

6. Horizontal distance = (6)(100.04 ft)$\left[\cos(\tan^{-1} 0.05)\right]$
= (600.24 ft)($\cos 2.8624^\circ$)
= 599.5 ft

7. a) Angular error = $\tan^{-1}\left[\frac{(34.5\ \cancel{ft})(2/\cancel{mile})}{5,280\ \cancel{ft/mile}}\right]$

= $\tan^{-1} 0.013$
= $0.749^\circ = 44.9'$

True declination = $21^\circ\ 30' - 00^\circ\ 45'$
= $20^\circ\ 45'$ E

b) In retracing the line from B to A he has, effectively, a 'declination' of $00^\circ 45'$ W. Therefore his compass bearing should be N $89^\circ\ 15'$ W.

8. Angle ABC = $180^\circ - (50^\circ\ 15' + 65^\circ\ 18')$
= $180^\circ - 115^\circ\ 33'$
= $64^\circ\ 27'$

By the Sine Law,

$$\frac{\sin 50^\circ\ 15'}{AC} = \frac{\sin 67^\circ\ 27'}{500\ ft}$$

(500 ft)($\sin 50^\circ\ 15'$) = (AC)($\sin 67^\circ\ 27'$)
(500 ft)(0.7688) = (AC)(0.9235)
384.4 ft = (AC)(0.9235)

$$\frac{384.4\ ft}{0.9235} = AC$$

416.24 ft = AC

Complete Solutions to B Problems (continued)

9. Correct distance = $(100 \text{ ft})\left[\cos\left(\tan^{-1}\frac{2.5 \cancel{\text{ft}}}{100 \cancel{\text{ft}}}\right)\right]$

$= (100 \text{ ft})(\cos 1.4321^\circ)$
$= (100 \text{ ft})(0.9997)$
$= 99.97 \text{ ft}$

Error = 100 ft - 00.07 ft = 0.03 ft

10. Slope distance = $\frac{100 \text{ ft}}{\cos(\tan^{-1} 0.6)}$

$= \frac{100 \text{ ft}}{\cos 30.96^\circ}$

$= \frac{100 \text{ ft}}{0.8575} = 116.6 \text{ ft}$

Elevation from abney to base of tree = (100 ft)(-140%) = - 140 ft.
Elevation from abney to top of tree = (100 ft)(15%) = 15 ft
Height of tree = 140 ft + 15 ft = 155 ft

11. HD = (slope distance)$\left[\cos(\tan^{-1}(\%\text{ grade}))\right]$

HD = $(100.04 \text{ ft})\left[\cos(\tan^{-1} 0.31)\right]$ = $(100.04 \text{ ft})(\cos 17.22^\circ)$
= $(100.04 \text{ ft})(0.9552)$
= 95.55 ft

HD = $(197.13 \text{ ft})\left[\cos(\tan^{-1} 0.27)\right]$ = $(197.13 \text{ ft})(\cos 15.1096^\circ)$
= $(197.13 \text{ ft})(0.9654)$
= 190.32 ft

HD = $(85.79 \text{ ft})\left[\cos(\tan^{-1} 0.59)\right]$ = $(85.79 \text{ ft})(\cos 30.5406^\circ)$
= $(85.79 \text{ ft})(0.8613)$
= 73.89 ft

12. DE = (950 ft - 520 ft)(0.15)
= (430 ft)(0.15) = 64.5 ft

$SD = \frac{DE}{\sin(\tan^{-1} 0.15)} = \frac{64.5 \text{ ft}}{\sin(\tan^{-1} 0.15)} = \frac{64.5 \text{ ft}}{\sin 8.5308^\circ}$

$= \frac{64.5 \text{ ft}}{0.1483} = 434.9 \text{ ft}$ or 435 ft

Complete Solutions to B Problems (continued)

13. A) Horizontal distance $= (200 \text{ ft})\left[\cos(\tan^{-1} 0.12)\right]$
$= (200 \text{ ft})(\cos 6.8428^\circ)$
$= (200 \text{ ft})(0.9929)$
$= 198.6 \text{ ft}$

b) Difference in elevation $= \frac{(H)(\% \text{ grade})}{100} = (198.6 \text{ ft})(0.12)$
$= 23.8 \text{ ft}$

14. Let L be the true slope distance and M be the measured slope distance.

$$L = \frac{937.5 \text{ ft}}{\cos(\tan^{-1} 0.32)} = \frac{937.5 \text{ ft}}{\cos 17.74^\circ} = \frac{937.5 \text{ ft}}{0.9524} = 984.33 \text{ ft}$$

$$M = \frac{(100 \text{ ft})(984.33 \text{ ft})}{99.95 \text{ ft}} = \frac{98{,}433 \text{ ft}}{99.95} = 984.8 \text{ ft}$$

15. a) Percent slope $= \tan\left(\cos^{-1}\left(\frac{HD}{SD}\right)\right)(100)$

$= \tan\left(\cos^{-1}\left(\frac{224 \text{ ft}}{235 \text{ ft}}\right)\right)(100)$

$= \tan(\cos^{-1} 0.9532)(100)$
$= (\tan 17.6^\circ)(100)$
$= (0.3172)(100)$
$= 31.72\%$

b) $DE = \frac{(HD)(\text{percent slope})}{100} = \frac{(224 \text{ ft})(31.72)}{100} = 71.1 \text{ ft}$

16. By the Cosine Law

$AB = \sqrt{(178.6 \text{ ft})^2 + (5{,}201.5 \text{ ft})^2 - 2(178.6 \text{ ft})(5{,}102.5 \text{ ft})\cos 93^\circ 16'}$

$= \sqrt{31{,}897.96 \text{ sq ft} + 26{,}035{,}506.25 \text{ sq ft} - (1{,}822{,}613 \text{ sq ft})(-0.057}$

$= \sqrt{26{,}067{,}404.21 \text{ sq ft} + 103{,}858.33 \text{ sq ft}}$

$= \sqrt{26{,}171{,}262.54 \text{ sq ft}}$

$= 5{,}115.79 \text{ ft}$

Complete Solutions to B Problems (continued)

16. Continued

By the Sine Law

$$\frac{\sin(CAB)}{178.6 \text{ ft}} = \frac{\sin 93^\circ 16'}{5{,}115.79 \text{ ft}}$$

$$\sin(CAB) = \frac{(\sin 93^\circ 16')(178.6 \text{ ft})}{5{,}115.79 \text{ ft}}$$

$$\sin(CAB) = \frac{(0.9984)(178.6)}{5{,}115.79}$$

$$\sin(CAB) = 0.03486$$
$$CAB = \sin^{-1} 0.03486$$
$$CAB = 2^\circ$$

Bearing of AB is N 87° 10' E

17. Slope distance $= \frac{-47.4 \text{ ft}}{\sin[\tan^{-1}(-0.275)]}$

$$= \frac{-47.4 \text{ ft}}{\sin(-15.3763^\circ)}$$

$$= \frac{-47.4 \text{ ft}}{-0.2652}$$

$$= 178.8 \text{ ft}$$

Convert to chains and links:

$$\frac{178.8 \text{ ft}}{66 \text{ ft/chain}} = 2 \text{ chains, } 46.8 \text{ ft}$$

$$\frac{46.8 \text{ ft}}{0.66 \text{ ft/link}} = 70.9 \text{ links}$$

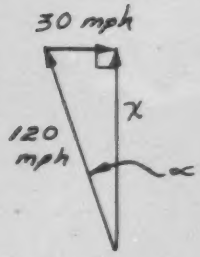
Slope distance = 2 chains 70.9 links

B. Problems without Solutions

1. If the destination of an airplane is due north 200 miles, the airplane flies at 120 mph, and there is a crosswind of 20 mph from due west, what heading should the airplane take and how long will it fly in order to reach its destination?

Complete Solutions to B Problems

1. From a vector diagram, we can find $\propto$ from



$\sin \propto = \frac{30}{120} = 0.25$

From a trig table, we get $\propto \approx 15^\circ$

Heading is $360^\circ - 15^\circ = 345^\circ$

Since the triangle is a right triangle, we can find x

$x^2 = 120^2 - 30^2 = 14{,}400 - 900 = 13{,}500$

$x = \sqrt{13{,}500} \approx 116$ mph

To go 200 miles at 116 mph: $\frac{200}{116} \approx 1.72$ hours

A. Problems with solutions.

1. In measuring the height of a tree on level ground a technician measures a point 1 chain (66ft) from the base of the tree. From that point the angle of elevation of the top of the tree is found to be $49^{\circ}30'$. Determine the height of the tree.

Solution:

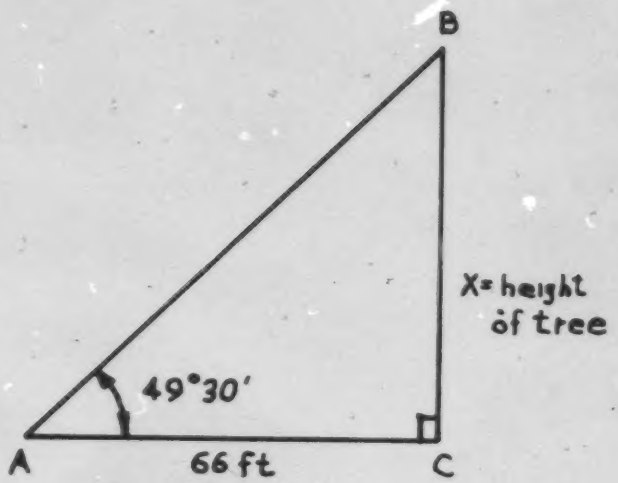
Since the tree is on level ground we assume that $\angle C$ is a right angle and that ACB is a right triangle.

$$\tan 49^{\circ}30' = \frac{x}{66 \text{ ft.}}$$

$$66 \text{ ft}(\tan 49^{\circ}30') = x$$

$$66 \text{ ft } (1.1708) = x$$

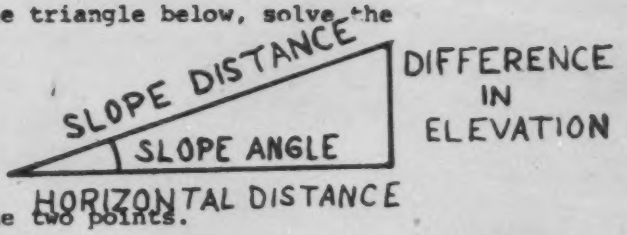
$$x = 77.3 \text{ ft.}$$



* A vertical angle read from the horizontal to a point higher is called an angle of elevation.

A. Problems with solutions, continued.

2. Using the definitions indicated on the triangle below, solve the following:

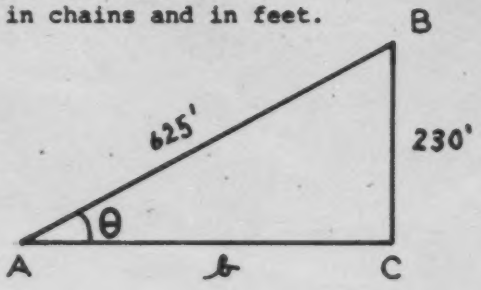


a) A slope distance of 625 feet is measured between two points that differ in elevation by 230ft. Determine the slope angle and also the horizontal distances between the two points.

b) If the slope distance between two stations is 5 chains and the slope angle is 5°40', determine the difference in elevation and the horizontal distances in chains and in feet.

Solution:

a) $\sin A = \frac{\text{oppside}}{\text{hyp}} = \frac{230 \text{ ft}}{625 \text{ ft}} = .3680$



$A = 21°36'$

$\cos 21°36' = \frac{b}{625 \text{ ft}}$

$b = 625 \text{ ft } (\cos 21°36') = 625 \text{ ft}(.9298) = 581.12 \text{ ft.}$

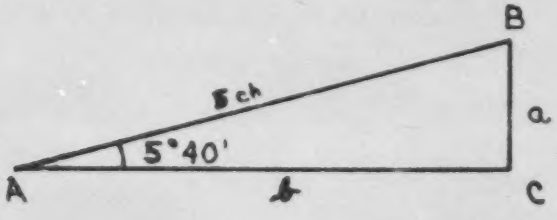
b) $\sin 5°40' = \frac{a}{5 \text{ chains}}$

$a = 5 \text{ chains } (\sin 5°40')$
$a = 5 \text{ chains } (.0987)$
$a = .494 \text{ chains}$
$a = (.494 \text{ chains}) \left(66 \frac{\text{ft}}{\text{ch}}\right) = 32.6 \text{ ft.}$

$\cos 5°40' = \frac{b}{5 \text{ chains}}$

$b = 5 \text{ chains } (\cos 5°40') = 5 \text{ chains } (.9951) = 4.98 \text{ chains}$

$b = (4.98 \text{ chains}) \left(66 \frac{\text{ft}}{\text{ch}}\right) = 328.4 \text{ ft.}$



B. Problems without solutions.

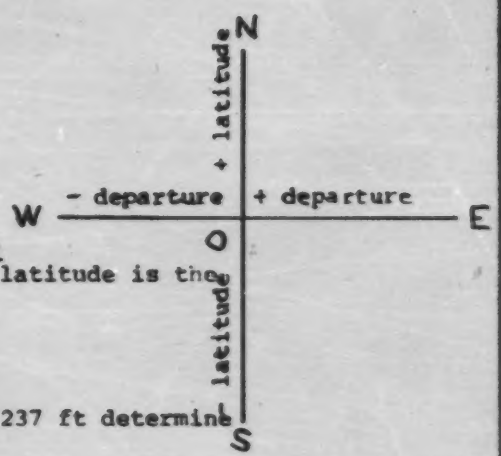
3. The sun's bearing (z) can be computed using the formula:

$$\cos z = \frac{\sin d - \sin h \sin l}{\cos h \cos l}$$ where d = declination
h = measured for refraction and parallax.
l = latitude (from map)

Determine z given d = 16°32.8'; h = 32°58.1'; l = 38°10.1'.

4. defn: Bearing is an acute angle measured from the north-south meridian while azimuth is an angle measured clockwise from the north.

defn: Using the north-south meridian as the y axis and the east-west line as the x axis of a coordinate system the departure is the x coordinate of a point while the latitude is the y coordinate.



a) Given a latitude of -85.4 ft and a departure of 237 ft determine bearing and azimuth.

b) Given an azimuth of 228 18' and a departure of -500 ft determine latitude.

B. Problems without complete solutions, continued.

4. continued

5. The base of a tree is inaccessable because of a rock obstruction near the tree. A transit is set up at random distance from the tree on level ground and an observation on the top of the tree gave an angle of elevation of $29^{\circ}40'$. A stake was set exactly two chains (1 chain = 66ft) farther away on the line of sight, and the angle of the top of the tree was now $21^{\circ}20'$. How high is the tree?

Complete solutions to B problems.

3. Solution:

$$\cos z = \frac{\sin 10^\circ 32.8' - (\sin 32^\circ 58.1')(\sin 38^\circ 10.1')}{(\cos 32^\circ 58.1')(\cos 38^\circ 10.1')}$$

$$\cos z = \frac{.28480 - (.54418)(.61797)}{(.83897)(.78620)}$$

$$\cos z = \frac{.28480 - .33629}{.65959}$$

$$\cos z = \frac{-.05149}{.65959} = -.07806$$

$$z = 94^\circ 28.6'$$

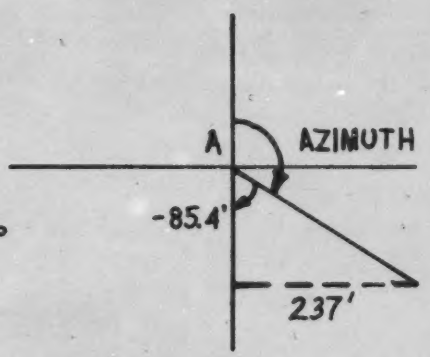
4. Solution:

a) $\tan A = \frac{237 ft}{85.4 ft} = 2.775$

$A = 70.2^\circ$

bearing = S70.2° E

azimuth = 180° - 70.2° = 109.8°



b) Bearing is S(228°18' - 180°)W

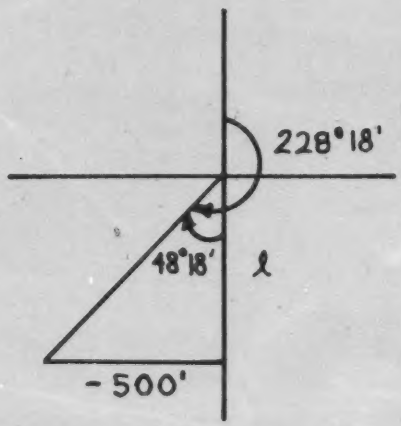
= S 48°18'W

$\text{ctn } 48^\circ 18' = \frac{l}{500 ft}$

$l = (500 ft)(\text{ctn } 48^\circ 18') = 500 ft (.89)$

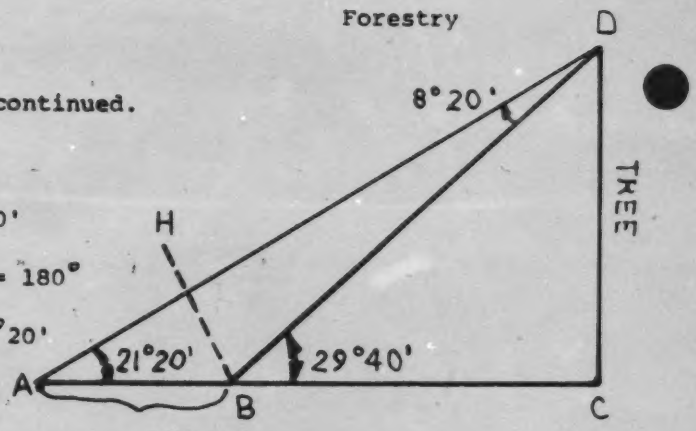
$l = 445.5 ft$

since in SE quadrant
latitude = -445.5 ft.



Complete solutions to B problems, continued.

5. Solution:



$\angle ABD = 180^\circ - 29^\circ 40' = 150^\circ 20'$

so $21^\circ 20' + 150^\circ 20' + \angle ADB = 180^\circ$

or $\angle ADB = 180^\circ - 21^\circ 20' - 150^\circ 20'$

$\angle ADB = 8^\circ 20'$

In order to use right triangle trig we construct right $\triangle AHB$ by drawing a perpendicular from B to AD. then

$\sin 21^\circ 20' = \frac{HB}{132 \text{ ft}}$ then in right $\triangle BHD$ $\sin 8^\circ 20' = \frac{48.02 \text{ ft}}{BD}$

$(.36379) = \frac{HB}{132 \text{ ft}}$

$(132 \text{ ft})(.36379) = HB$

$48.02 \text{ ft} = HB$

$BD (\sin 8^\circ 20') = 48.02 \text{ ft}$

$BD (.14493) = 48.02 \text{ ft.}$

$BD = \frac{48.02 \text{ ft}}{.14493}$

$BD = 331.33 \text{ ft}$

then in rt. $\triangle BCD$ $\sin 29^\circ 40' = \frac{DC}{331.33 \text{ ft}}$

$(331.33 \text{ ft})(\sin 29^\circ 40') = DC$

$(331.33 \text{ ft})(.49495) = DC$

$164 \text{ ft.} = DC$

A. Problems with complete solutions.

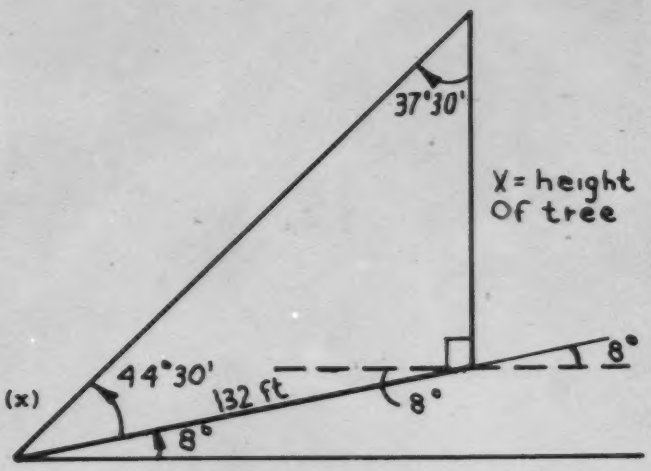
1. A tree of unknown height is on a slope which has been determined to be 8°. A distance of 2 chains (132 ft) is measured from the base of the tree in a downhill direction. From that point the top of the tree is sighted and the line of sight makes a $44^\circ\ 30'$ angle with the ground. Determine the height of the tree.

Solution:

Since the slope is 8°, the tree makes an angle of $90^\circ + 8^\circ = 98^\circ$ with the hill (downhill). The third angle of the triangle is then:

$180^\circ - 98^\circ - 44^\circ\ 30' = 37^\circ\ 30'$

Since we know all three angles and one side we can now apply the law of sines:



$$\frac{132\text{ft}}{\sin 37^\circ 30'} = \frac{x}{\sin 44^\circ 30'}$$

$$\frac{132\text{ ft}}{.60876} = \frac{x}{.70091}$$

$$(132\text{ ft})\ (.70091) = (.60876)\ (x)$$

$$\frac{(132\text{ ft})(.70091)}{160676} = x$$

$$x = 152\text{ ft.}$$

B. Problems without Solutions

2. Two fire stations are located 8.4 miles apart with tower B directly north of tower A. A fire is spotted bearing S74° 36'W from B and bearing S82° 17'W from A. How far is the fire from tower A?

3. On a slope of 15° a tree of unknown height is to be measured. A distance of 1 chain (66 ft) is measured in an uphill direction. From that point the line of sight to the top of the tree makes an angle of 34° 20' with the horizontal. Determine the height of the tree.

Complete solutions to B problems.

2. Solution:

The angle within the triangle at A is $180^\circ - 82^\circ 17' = 97^\circ 43'$. Then the angle at F becomes $180^\circ - 97^\circ 43' - 74^\circ 36' = 7^\circ 41'$

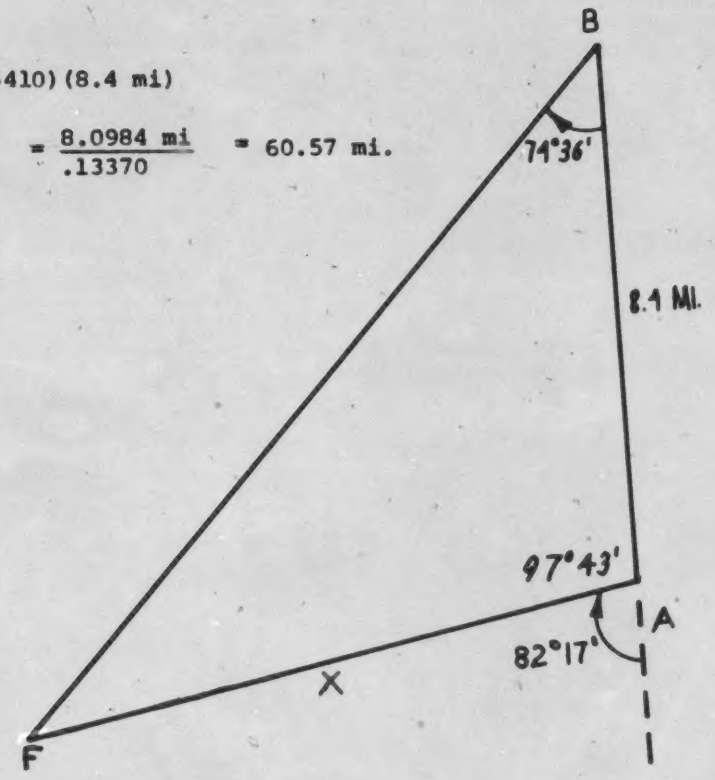
then applying the Law of Sines:

$$\frac{x \text{ mi}}{\sin 74^\circ 36'} = \frac{8.4 \text{ mi}}{\sin 7^\circ 41'}$$

$$\frac{x \text{ mi}}{.96410} = \frac{8.4 \text{ mi}}{.13370}$$

$$(.13370)(x \text{ mi}) = (.96410)(8.4 \text{ mi})$$

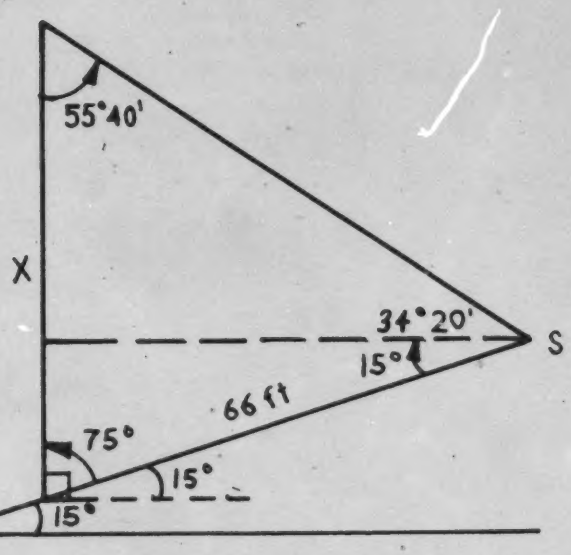
$$x \text{ mi} = \frac{(8.4 \text{ mi})(.96410)}{.13370} = \frac{8.0984 \text{ mi}}{.13370} = 60.57 \text{ mi.}$$



Complete Solutions to B problems.

3. Solutions:

Since the slope is 15° the tree makes an angle with the hill (uphill) of 90° -15°=75°and the angle as S becomes 34°20' + 15°= 49°20'. The remaining angle of the triangle is then 180°- 49°20' - 75°= 55°40'.



Then applying the Law of Sines

$$\frac{x}{\sin 49°20'} = \frac{66 \text{ ft}}{\sin 55°40'}$$

$$\frac{x}{.75851} = \frac{66 \text{ ft}}{.82577}$$

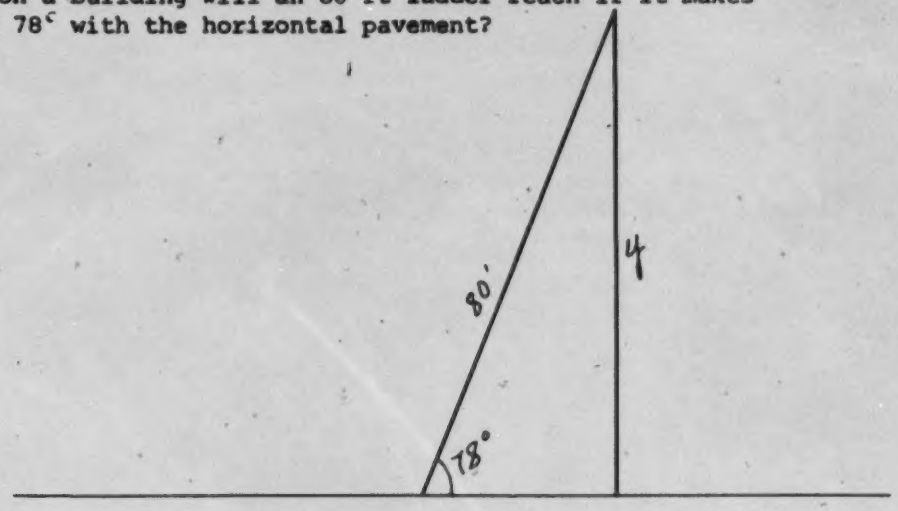
$$(.82577)\ (x) = (.75851)\ (66\text{ft})$$

$$x = \frac{66\text{ft}\ (.75851)}{.82577}$$

$$x = \frac{50.062 \text{ ft}}{.82577} = 60.6 \text{ ft.}$$

A. Problems with Solutions

1. How far up on a building will an 80 ft ladder reach if it makes an angle of 78° with the horizontal pavement?



Solution:

You need to determine what trig ratio to use with the ratio y/80 ft. Since, in relation to the angle 78°, we have the ratio of opposite side to hypotenuse, we should use the sine.

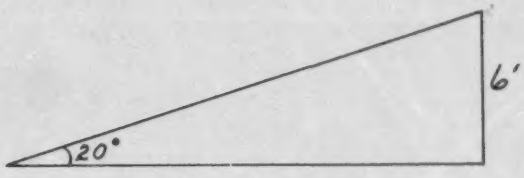
$$\sin 78^\circ = \frac{y}{80 \text{ ft}}$$

$$0.9781 = \frac{y}{80 \text{ ft}}$$

$$78.248 \text{ ft} = y$$

So the ladder will reach 78.25 feet or 78 ft 3 in up the wall.

2. A bullet is found imbedded in the wall of a room. The vertical distance from the floor to the bullet is 6 ft. The path of the bullet in the wall is inclined upward at an angle of 20° with the horizontal. How far from the wall was the gun at the time of the firing if the bullet was fired (a) from the floor? (b) three feet above the floor?



A. Problems with Solutions (continued)

2. (continued)

Solution:

You need to determine the base length of each right triangle.

(a) $\cot 20^\circ = \frac{x}{6 \text{ ft}}$

$2.747 = \frac{x}{6 \text{ ft}}$

From the floor, the gun was fired at a distance of 16.5 feet or 16 ft 6 in.

$16.482 = x$

(b) $\cot 20^\circ = \frac{x}{3 \text{ ft}}$

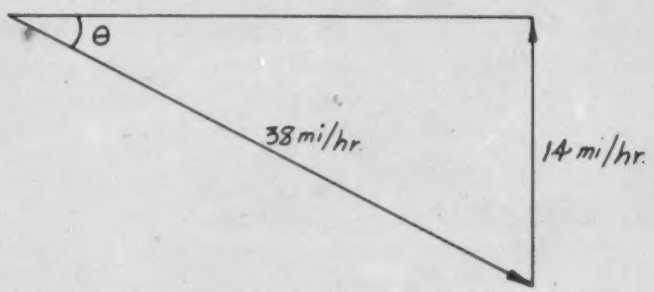
$2.747 = \frac{x}{3 \text{ ft}}$

From the height of three feet above the floor, the gun was fired from a distance of 8.24 feet or 8 ft 3 in.

$8.241 \text{ ft} = x$

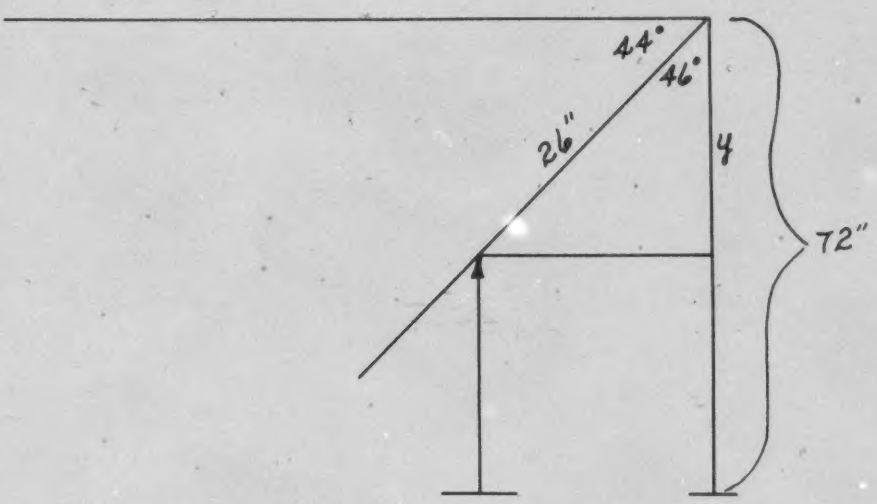
B. Problems without Solutions

3. A boat travels at 38 miles/hr in still water. At what angle with the shore must the boat be steered to reach a point directly opposite if the velocity of the current is 14 miles/hr?



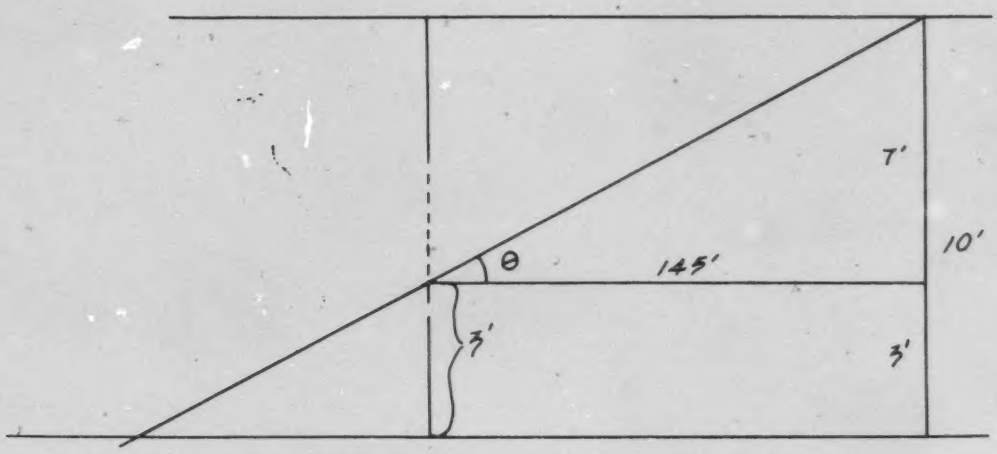
B. Problems without Solutions (continued)

4. An axe cut is found on the wall of a room. It is at an angle of 44° with the horizontal, and the lower end is 72 in above the floor. Assuming the axe handle to be 26 in long, how far above the floor was the lower end of the handle when the cut was made?



B. Problems without Solutions (continued)

5. A bullet was imbedded in the ceiling after being fired into a room through a window. At what angle was the gun fired if the bullet hole in the glass was 3 ft above the floor, and the horizontal distance from the imbedded bullet to the window glass was 14.5 ft? The distance from ceiling to floor was 10 ft.



Complete Solutions to B Problems (continued)

3. You need to set up a ratio between the two known sides of the right triangle, and then find the correct trig ratio to use with those sides.

$\tan \theta = \frac{14}{38}$

$\tan \theta = 0.3684$ now use a trig table to find the angle

$\theta = 20^{\circ}$

4. You need to find the "y" distance in the drawing, and then subtract that from 72 inches.

$\cos 46^{\circ} = \frac{y}{26 \text{ inches}}$

$0.6947 = \frac{y}{26 \text{ inches}}$ multiply both sides by 26 inches

18.06 inches = y

72 inches - 18.06 inches = 53.94 inches

So the lower end of the handle was about 54 inches above the floor.

5. You need to select the correct trig ratio using the ratio of the sides 7 ft and 14.5 ft in relation to the angle we are trying to find.

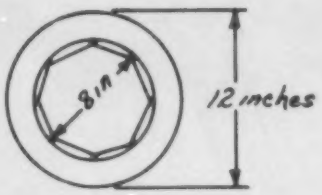
$\tan \theta = \frac{7 \cancel{ft}}{14.5 \cancel{ft}}$

$\tan \theta = 0.4828$

$\theta = 25.77^{\circ}$ The gun was fired from an angle of about 26°

A. Problems with Solutions

1. The job is to drill and tap 8 holes on the 8 inch bolt circle of a 12 inch cover. Find the distance between each hole center to center dimension.



Solution:

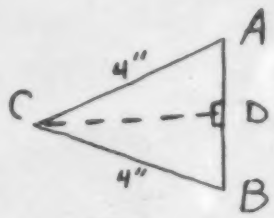
In the above figure, we wish to find the distance AB. Draw a triangle using points A, B, and the center of the circle C, as shown

Each side, AC and BC, is 4". Angle ACB is 45°, so angle ACD is 22 1/2°.

Therefore,

$$\sin 22\ 1/2^{\circ} = \frac{AD}{4"}$$

or $$0.38268 = \frac{AD}{4"}$$



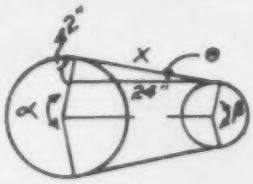
1.53072" = AD and since AB = (2)(AD)

AB = (2)(1.53072") = 3.06144 in $\approx$ 3.061 in

Alternate Solution:

According to a table, the desired distance S is equal to 0.383 C where C is the "distance across corners," in this case, 8 inches. So, S =(0.383)(8 in) = 3.064 in

2. If a 10-inch radius belt pulley and an 8-inch radius belt pulley are centered 24 inches apart, how long should the belt be?



A. Problems with Solutions (continued)

2. (continued)

Solution:

An approximate solution will be to find the sum of 2x (see sketch) and that part of each circumference contacted by the belt, namely $\left(\frac{\alpha}{360}\pi\right)(20 \text{ in}) + \left(\frac{\beta}{360}\pi\right)(16 \text{ in})$ where α and β are the angles in degrees.

By the Pythagorean theorem, $x^2 = (24 \text{ in})^2 - (2 \text{ in})^2 = 572 \text{ sq in}$

So, $x = \sqrt{572 \text{ sq in}} \approx 23.9 \text{ in}$ and $2x = 47.8 \text{ in}$

$\sin \theta = 2 \text{ in}/24 \text{ in} = 1/12 = 0.0825, \quad \theta \approx 5$

It follows that $\alpha \approx 190^\circ$ and $\beta \approx 170^\circ$

Substituting into the expression above,

$$\left[\left(\frac{190}{360}\right)(3.14)\right]\left[20 \text{ in}\right] + \left[\left(\frac{170}{360}\right)(3.14)\right]\left[16 \text{ in}\right]$$

$$\left(\frac{3.14}{360}\right)\left[(190)(20 \text{ in}) + (170)(16 \text{ in})\right]$$

$$\left(\frac{3.14}{360}\right)(3800 \text{ in} + 2720 \text{ in})$$

$$\left(\frac{3.14}{360}\right)(6520 \text{ in}) \approx 56.87 \text{ in}$$

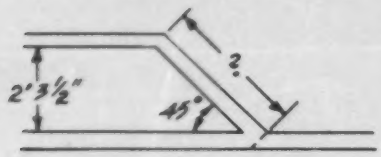
So 47.8" + 56.87" = 104.67 in

B. Problems without Solutions

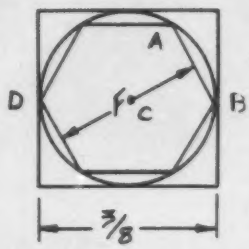
3. Find the height of a crane boom if the angle of elevation from the observer is 60° when measured 160 feet from a point directly below the end of the boom.

B. Problems without Solutions (continued)

4. Installing a pipeline, a mechanic has to run a 45° offset. The perpendicular distance between the two pipelines is 2 ft 3 1/2 in. What length of pipe will he need for the connection between the two lines? (Note: Subtract 1 inch from the computed length to allow for the fittings on the two ends.)



5. It is desired to mill a hexagon on bar stock with a diameter of 3/8 inch. What is the distance across the flats if we assume that he wants the hexagon as large as possible.



Complete Solutions to B Problems

3. $\tan 60^\circ = \frac{h}{160 \text{ ft}}$ so $h = (160 \text{ ft})(\tan 60^\circ)$

$= (160 \text{ ft})(1.732)$

$= 277.120 \text{ ft}$

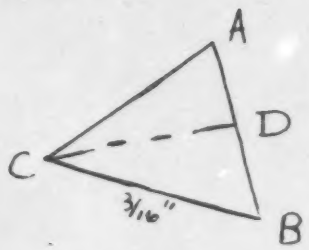
4. $\sin 45^\circ = \frac{27.5 \text{ in}}{x}$

$x = \frac{27.5 \text{ in}}{\sin 45^\circ}$

$= \frac{27.5 \text{ in}}{0.7071}$

$= 38.9 \text{ in}$ So, $38.9 - 1 = 37.9 \text{ in}$

5. In the figure for problem 5, we wish to find the distance F. Draw a triangle using points A, B, and the center of the hex C, as shown:



CD = 1/2 F

BC = 1/2 of 3/8" = 3/16"

Angle ACB is 60°, so angle ACD is 30°

Therefore, $\cos 30^\circ = \frac{1/2 \text{ F}}{3/16 \text{ in}}$

or $0.8660 = \frac{1/2 \text{ F}}{3/16 \text{ in}}$

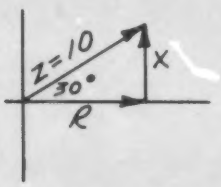
$(0.8660)(3/16 \text{ in}) = 1/2 \text{ F}$

$0.1624 \text{ in} = 1/2 \text{ F}$

So, $0.3248 \text{ in} = \text{F}$

A. Problems with Solutions

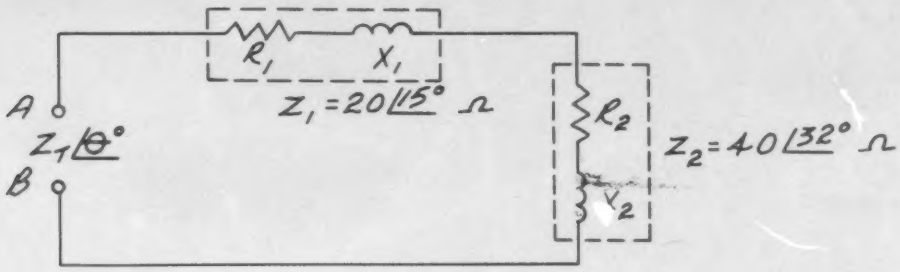
1. Impedance Z is a complex quantity involving both magnitude Z and direction θ. For instance, $Z = 10 \angle 30^\circ \Omega$ is graphically represented below.



Remember:
1. The horizontal axis component is resistive.
2. The vertical axis component is reactive.

When combining impedances, it is necessary to break them down into their resistive and reactive elements; then, combine $R_1 + R_2$, $X_1 + X_2$ and then, express the resulting impedance as $Z_T \angle \theta^\circ$

Given:



(a) Find R_1, R_2, X_1, X_2

(b) Calculate $Z_T \angle \theta^\circ$

Solution:

(a) $Z_1 = 20 \angle 15^\circ$: $R_1 = 20 \cos 15^\circ = 19.3 \Omega$ $R_2 = 40 \cos 32^\circ = 33.9 \Omega$
$X_1 = 20 \sin 15^\circ = 5.18 \Omega$ $X_2 = 40 \sin 32^\circ = 29.2 \Omega$

$R_1 = 19.3 \Omega$, $R_2 = 33.9 \Omega$, $X_1 = 5.18 \Omega$, $X_2 = 29.2 \Omega$

(b) $R_1 + R_2 = 19.3 + 33.9 = 53.2 \Omega$

$X_1 + X_2 = 5.18 + 29.2 = 34.4 \Omega$

$$\theta = \text{Tan}^{-1} \frac{34.4}{53.2} = 32.9^\circ$$

$$Z_T = \frac{R_T}{\cos \theta} = \frac{53.2}{\cos 32.9^\circ} = 63.4 \Omega$$

$$Z_T \angle \theta^\circ = 63.4 \angle 32.9^\circ \Omega$$

B. Problems without Solutions

2. A television technician is to install a 50-ft telescoping antenna mast on a flat roof. The highest guy wire connect point on the mast is 40 ft above the roof surface. If the safety specifications require a minimum angle of 30° between the mast and guy wires, and there are 4 guy wires at each level, (a) what is the minimum distance between the anchor bolts on the roof for the 40-ft level guy wires? (b) What is the length of the individual 40-ft level guy wires?

B. Problems without Solutions (continued)

3. Impedance consists of a "resistive" effect Z but also introduces a phase shift θ between an applied voltage and the resulting current. Both Z and θ are functions of the circuit components and, for a given arrangement, vary only with the frequency of the applied AC voltage. Impedance can be expressed as a vector (phasor) in the form: $\bar{Z} = Z \angle \theta^{\circ}$ where Z is the magnitude and θ is the phase shift angle of the impedance.

Impedances for the three basic components are:

Resistance: $\bar{Z}_R = R \angle 0^{\circ}$

Inductance: $\bar{Z}_L = 2\pi fL \angle 90^{\circ}$

Capacitance: $\bar{Z}_C = \frac{1}{2\pi fC} \angle -90^{\circ}$

f = Hertz
L = Henrys
C = Farads
Z = OHMS

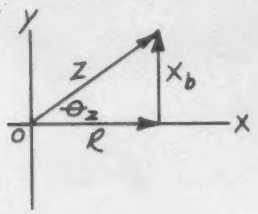
Impedance phasors combined with current and voltage phasors obey OHM's Law in all respects.

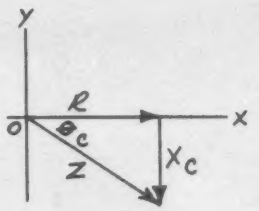
(a) Find $\bar{Z}_R$ of a 30 Ω resistor, $\bar{Z}_L$ of a 5 millihenry coil and $\bar{Z}_C$ of a 0.002 microfarad capacitor for a frequency of 50 KHertz.

(b) Plot the impedance Z_T resulting if these three components were hooked in series.

B. Problems without Solutions (continued)

4. Inductive reactance (X_L) and capacitive reactance (X_C) affect alternating current curcuits in a fashion similar to resistance in direct current circuits. That is, X_C and X_L limit the AC current flow for a given applied AC voltage. A major difference between reactance and resistance exists in that reactors never consume power. This is due to a 90° phase shift between the applied voltage and the resulting current, producing a product of VI equal to zero. Inductive reactance causes the current to lag the applied voltage while capacitive reactance causes the current to lead the applied voltage, in both cases by 90°. Resistors, which cause no phase shift, are often combined with X_L and/or X_C in AC circuits. The resulting complex current-inhibiting action is called impedance (Z) and consists of both magnitude and direction as illustrated below. Note that the angle between resistance and reactance is +90° for X_L and -90° for X_C.





(a) Given a circuit with $X_L = 250\,\Omega$ and $R = 115\,\Omega$, find Z and θ_L.

(b) Given a circuit with $X_C = 360\,\Omega$ and $R = 720\,\Omega$, find Z and θ_C.

B. Problems without Solutions (continued)

5. The general expression for a voltage sine wave is $v = Vm \sin(wt+\theta)$. This represents a voltage of peak value Vm generating a sinusoidial pattern at the rate wt. Since the wave is symetrical and repeating, it can be used in steady state circuit work as a standing vector of magnitude V at angle θ. V is the DC equivalent of the voltage, permitting the use of OHM's and Kirchoff's law in AC applications and is equal to $\frac{Vm}{\sqrt{2}}$ or 0.707 Vm. θ is the phase angle of the wave relative to any referenced axis and is a constant. The vector is expressed $V = V \angle \theta$ and is called a PHASOR. Current sinusoids are treated in exactly the same manner.

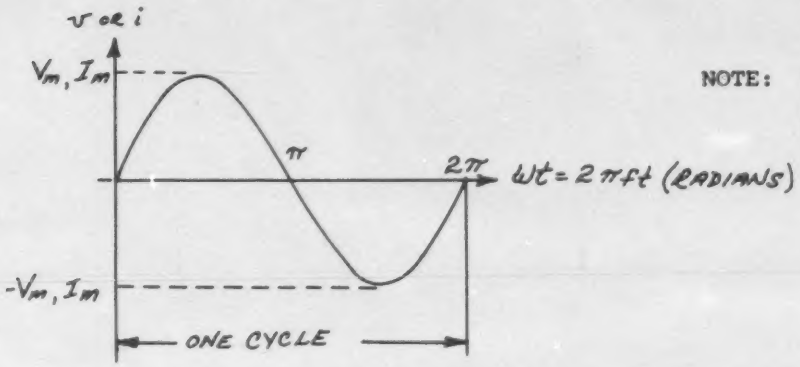
   If a tape recorder was connected to a voltage $v = 166 \sin(377t + 0^{\circ})$ and a current $1 = 1.5 \sin (377t + 25^{\circ})$ was to flow,

   (a) Express the voltage as a Phasor.
   (b) Express the current as a Phasor.
   (c) Calculate the power used by the recorder.

   Ref: 1. Power = VI Cos θ watts (when θ = phase angle between V and I)
   2. $w = 2\pi$ x frequency
   3. t = time in sec

B. Problems without Solutions (continued)

6. The most common AC wave form encountered in electronics is the Sine wave. Its general forms are: v = Vm Sin wt (voltage) and I = Im Sin wt (current), where v and I are instantaneous values of voltage and current, Vm and Im are the peak excursion from zero, w = 2 π x frequency in Hertz and t = time in seconds.



NOTE: Hertz = cycles per second

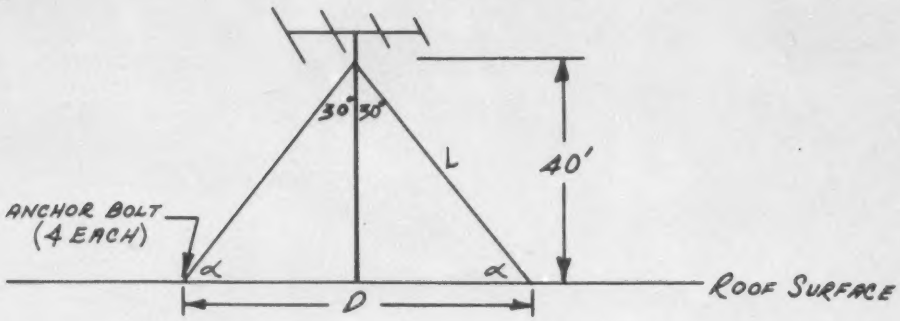
The expression for the 60 Hertz voltage in the plugs in your home outlets is v = 166 Sin 377T.

(a) Find the time for one cycle of this voltage.

(b) What is the value of the voltage at t = 11.5 millisec?

Complete Solutions to B Problems

2.



(a) $90^\circ + 30^\circ + \alpha = 180^\circ \quad \alpha = 60^\circ$

$$\text{Tan } \alpha = \frac{40'}{D/2} = \frac{80'}{D} \; ; D = \frac{80'}{\text{Tan } \alpha} = \frac{80'}{\text{Tan } 60^\circ} = \frac{80'}{1.7321} = 46.2 \text{ feet}$$

D = 46.2 feet

(b) $$\text{Sin } \alpha = \frac{40'}{L} \; ; \; L = \frac{40'}{\text{Sin } \alpha} = \frac{40'}{\text{Sin } 60^\circ} = 46.2 \text{ feet}$$

L = 46.2 feet

Alternate Solution: Since this is obviously an equilateral triangle, once D is found, L is immediately defined.

3. Finding individual impedances:

(a) $\bar{Z}_r = R \angle 0^\circ = 30 \angle 0^\circ$

$\bar{Z}_L = 2\pi fL \angle 90^\circ = 2\pi \times 50 \times 10^3 \times 5 \times 10^{-3} \angle 90^\circ = 1571 \angle 90^\circ$ OHMS

$$\bar{Z}_c = \frac{1}{2\pi fC} \angle -90^\circ = \frac{1}{2\pi \times 50 \times 10^3 \times 0.002 \times 10^{-6}} \angle -90 = 1592 \angle -90^\circ \text{ OHMS}$$

(b) Plot the sum of the vectors above, $\bar{Z}_r + \bar{Z}_L + \bar{Z}_c$

Since $\bar{Z}_L$ and $\bar{Z}_c$ are opposing,

$\bar{Z}_c - \bar{Z}_L = 1592 \angle -90^\circ - 1571 \angle 90^\circ = 21 \angle -90^\circ$

Plotting $30 \angle 0^\circ$ against $21 \angle -90^\circ$ yields:

$\bar{Z}_T = 36.6 \angle -35^\circ$

Complete Solutions to B Problems (continued)

4. (a) $\theta_L = \text{Tan}^{-1}\left(\frac{X_L}{R}\right) = \text{Tan}^{-1}\left(\frac{250}{115}\right) = 65.3^\circ$

$$Z = \frac{R}{\text{Cos } \theta_L} = \frac{X_L}{\text{Sin } \theta_L} = \frac{115}{\text{Cos } 65.3^\circ} = \frac{250}{\text{Sin } 65.3^\circ} = 275\ \Omega$$

(b) $\theta_C = -\text{Tan}^{-1}\left(\frac{X_C}{R}\right) = \text{Tan}^{-1}\left(\frac{360}{720}\right) = -26.6^\circ$

$$Z = \frac{R}{\text{Cos } \theta_C} = \frac{X_C}{\text{Sin } \theta_C} = \frac{720}{\text{Cos}(-26.6^\circ)} = \frac{360}{\text{Sin}(-26.6^\circ)} = 805\ \Omega$$

5. (a) Find the equivalent DC voltage $V = \frac{166}{\sqrt{2}} = 117$ Volts

Express the voltage as a Phasor $V = 117 \angle 0^\circ$ Volts

(b) Find the equivalent DC current $I = \frac{1.5}{\sqrt{2}} = 1.06$ Amps

Express the current as a Phasor $\bar{I} = 1.06 \angle 25^\circ$ Amps

(c) $P = VI \text{ Cos } \theta = (117 \text{ volts} \times 1.06 \text{ Amps}) \text{ Cos } 25^\circ = 112.4$ Watts

P = 112.4 Watts

6. (a) Beginning at t = 0, one cycle is complete when 2π ft = 2π radians. Solving for t:

$$t = \frac{2\pi}{2\pi f} = \frac{1}{f} = \frac{1}{60} = 0.0167 \text{ sec}$$

t = 16.7 millisec
Note that the time T for 1 cycle of any repetitive Sine wave is $T = \frac{1}{f}$

(b) $v = 166 \text{ Sin } 377t$
Substituted 11.5 millisec for t:
$v = 166 \text{ Sin } (577 \times 11.5 \times 10^{-3}) = 166 \text{ Sin } (4.336 \text{ radians})$
$= 166 \text{ Sin } (4.336 \times 57.3^\circ) = 166 \text{ Sin } 248^\circ$

248° is in the 3rd quadrant, therefore,
$v = 166 \text{ Sin } (270^\circ - 248^\circ) = -166 \text{ Sin } (22^\circ) = -62.2$ Volts

A. Problems with solutions.

1. How high up the side of a building will a 24 ft extension ladder reach when set at a recommended safety angle of 75° with the ground?

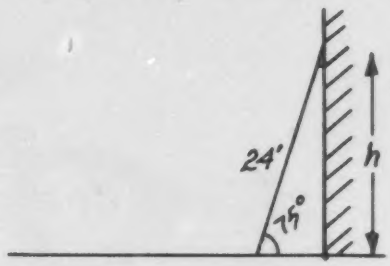
Solution:

$$\sin 75^\circ = \frac{h}{24 \text{ ft.}}$$

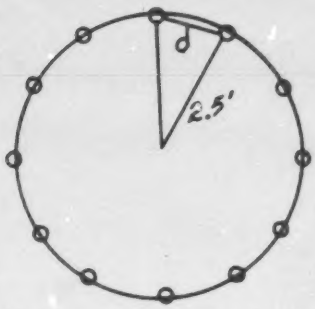
$$(24 \text{ ft.})\ (\sin 75^\circ) = h$$

$$(24 \text{ ft.})\ (.9059) = h$$

$$23.2 \text{ ft.} = h$$



2. Find the center to center distance between 12 bolt holes drilled on the rim of a 2.5 foot radius circle.



$$\frac{360^\circ}{12} = 30^\circ$$

Using Law of Cosines:

$$d = \sqrt{a^2 + b^2 - 2\ ab\ \cos D}$$

$$d = \sqrt{(2.5 \text{ ft.})^2 + (2.5 \text{ ft.})^2 - 2(2.5 \text{ ft.})\ (2.5 \text{ ft.})\ (.8660)}$$

$$d = \sqrt{6.25 \text{ ft.}^2 + 6.25 \text{ ft.}^2 - 10.825 \text{ ft.}^2}$$

$$d = \sqrt{1.675 \text{ ft.}^2}$$

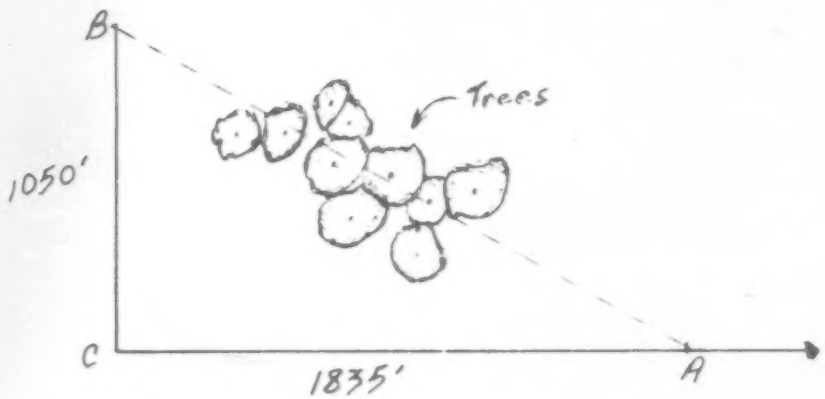
$$d = (1.29 \text{ ft.}) \left(\frac{12 \text{ in}}{1 \text{ ft.}}\right) = 15.5 \text{ in.}$$

B. Problems without solutions.

3. How long a rafter is required for a shed roof whose span is 30 ft. if the roof makes an angle of 24° with the horizontal?

4. Determine how far up a hillside a tank must be located to provide a 80-foot minimum head (vertical height) of water at a valve located at the base of the hill, the average slope of the hill is 27°?

5. It is necessary to run a fence line from point A to point B, in the figure below, neither point being visible from the other. At what angle should the fence line be set in relation to line AC?



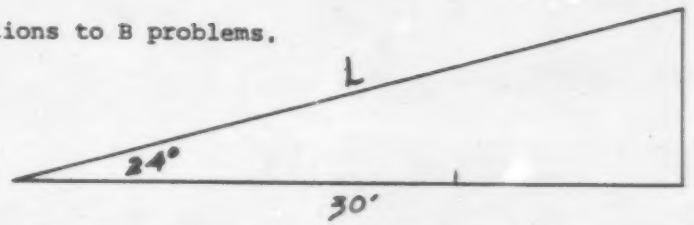
B. Problems without solutions, continued.

6. A guy wire from the top of a pole to a point on the ground 35 ft. from the foot of the pole makes an angle of 77° from the horizontal. a) How high is the pole? b) How long must the guy wire be if two feet of wire are needed for tying?

7. Find the center to center distance between 8 bolt holes drilled equidistant in the rim of a 4 foot circle.

Complete solutions to B problems.

3.

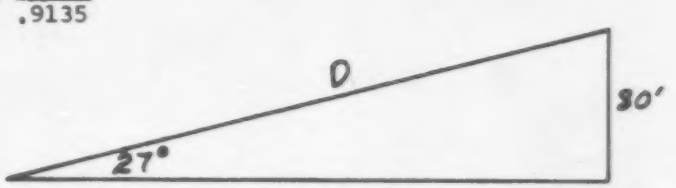


$$\cos 24^\circ = \frac{30 \text{ ft.}}{L}$$

$$L = \frac{30 \text{ ft.}}{\cos 24^\circ} = \frac{30 \text{ ft.}}{.9135}$$

$$L = 32.84 \text{ ft.}$$

4.



$$\sin 27^\circ = \frac{80}{D}$$

$$D = \frac{80}{\sin 27^\circ} = \frac{80}{.4540} = 176 \text{ ft. approx.}$$

5.

$$\tan A = \frac{1050 \text{ ft.}}{1835 \text{ ft.}}$$

$$\tan A = 0.5722$$

$$A = 30^\circ \text{ approx.}$$

6.

$$\tan 77^\circ = \frac{H}{35 \text{ ft.}}$$

$$(35 \text{ ft.})\ (\tan 77^\circ) = H$$

$$(35 \text{ ft.})\ (4.3315) = H$$

$$151.6 \text{ ft.} = H$$

$$\cos 77^\circ = \frac{35 \text{ ft}}{D}$$

$$D = \frac{35 \text{ ft}}{\cos 77^\circ}$$

$$D = \frac{35 \text{ ft}}{.2250}$$

$$D = 155.6 \text{ ft.}$$

Complete solutions to B problems, continued.

7.

$$\frac{360^{\circ}}{8} = 45^{\circ}$$

Using Law of Cosines:

$$d = \sqrt{a^2 + b^2 - 2ab\cos D}$$

$$d = \sqrt{(2\text{ ft.})^2 + (2\text{ ft.})^2 - 2(2\text{ ft.})(2\text{ ft.})(.707)}$$

$$d = \sqrt{4\text{ ft.}^2 + 4\text{ ft.}^2 - 5.656\text{ ft.}^2}$$

$$d = \sqrt{2.344\text{ ft.}^2}$$

$$d = (1.53\ \cancel{\text{ft.}})\left(\frac{12\text{ in.}}{\cancel{\text{ft.}}}\right) = 18.4\text{ in.}$$